STATISTICS 117: Mathematics for Statistics
Exercises for Review

I. Write TRUE if the statement is always true. Otherwise, write FALSE.

Let p, q and r be propositions.

1. $\sim (p \vee q) \rightarrow \sim q$
2. $(p \wedge q) \rightarrow r \equiv p \rightarrow (r \vee \sim q)$

Let x, y be real numbers.

3. A sufficient condition for $x^2 - 2x + 1 = 0$ is $x = 1$.
4. A necessary condition for $x^2 - 2x + 1 = 0$ is $x = 1$.
5. $x \leq y$ and $x \geq y$ is a contradiction.
6. $xy = 0$ iff $x = 0$ or $y = 0$.
7. $xy \neq 0$ iff $x \neq 0$ or $y \neq 0$.

Let A = set of even integers and B = $\mathbb{Z}$.

8. $\exists x \in A, \forall y \in B$ such that $x = 2y$.
9. $\forall x \in A, \exists y \in B$ such that $x \neq 2y$.
10. $\forall x \in B, \exists y \in B$ such that $x = 2y$.

11. If $3 > 7$, then $6 < 14$.
12. If $4 = 8$, then $2 > 4$.
13. $(A \wedge B) \rightarrow C$
    $/ \therefore A \rightarrow C$
14. An if-then statement is logically equivalent with its contrapositive.
15. The converse and inverse of an if-then statement is biconditional.

II. Construct the truth table for the following proposition.

1. $((p \wedge q) \vee \sim r) \leftrightarrow (q \rightarrow r)$
2. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
3. $(\sim p \wedge r) \rightarrow q \equiv ((q \wedge r) \wedge \sim p) \rightarrow r$

III. Prove the validity of the following arguments using rules of inference.

1.
$Q \vee (R \wedge S)$
$Q \rightarrow S$
$/ \therefore S$

2.
$A \rightarrow (B \wedge C)$
$(C \vee D) \rightarrow E$
$/ \therefore A \rightarrow E$

3.
$(p \wedge q) \equiv r$
$\sim (r \vee \sim p)$
$/ \therefore \sim q$

4. If you are young, then you are restless and gullible.
   If you are restless or you are gullible then you often make mistakes.
   Therefore, if you are young, then you often make mistakes.

5. If Barney loves Robin or Ted admires Robin, then Ted will not be marrying Robin. A sufficient condition for Barney marrying Robin is Ted will not marry Robin or Ted will not find the Mother. Barney loves Robin. Ted finds the Mother. Therefore, Barney will marry Robin.

6. Let the domain of discourse be the set of all students.
   All intelligent Stat majors will become rich.
   All lazy students are intelligent but will not become rich.
   Therefore, all Stat majors are not lazy.

7. No mathematicians are historians. Some mathematicians are scientists. Therefore some scientists re not historians.

IV. Disprove the following.

1. Let the domain of discourse be the set of whole numbers.
   $\forall x, \forall y, x + y > 0$
2. Let p, q and r be propositions.
   $(p \vee q) \rightarrow (q \wedge r)$

END OF EXERCISES