One Population Case

Table 1. Tests Concerning the Population Mean

Ho: $\mu = \mu_o$

(where μ_o is a specified hypothesized value of the population mean)

Test Statistic	Ha	Region of Rejection
Case 1: σ known $Z_c = \dfrac{\bar{x} - \mu_o}{\sigma / \sqrt{n}}$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \neq \mu_o$	$z_c < -z_\alpha$ $z_c > z_\alpha$ $z_c < -z_{\alpha/2}$ & $z_c > z_{\alpha/2}$
Case 2: σ unknown $t_c = \dfrac{\bar{x} - \mu_o}{s / \sqrt{n}}$	$\mu < \mu_o$ $\mu > \mu_o$ $\mu \neq \mu_o$	$t_c < -t_{(\alpha, v)}$ $t_{c} > t_{(\alpha, v)}$ $t_c < -t_{(\alpha/2, v)}$ & $t_c > t_{(\alpha/2, v)}$ $v = n-1$

- The above tests are exact α-level tests for samples from a normal distribution. However, they provide good approximate α-level test when the distribution is not normal provided that the sample size is n>30.

- Case 3: If σ^2 is unknown and n>30, use the z-test but replace σ by s, that is,

$$Z_c = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Note: tabulated z-values for the common choices of α

α	0.01	0.05	0.10
z_{α}	2.33	1.645	1.28
$z_{\alpha/2}$	2.576	1.96	1.645

Example: One Tail Test

Q. Does an average box of cereal contain more than 368 grams of cereal? A random sample of 25 boxes showed $\bar{X}$ = 372.5 grams. The company has specified σ to be 15 grams. Test at the $\alpha =$ 0.05 level of significance.

H_0: $\mu = 368$ grams
Ha: $\mu > 368$ grams
where μ = true mean content of cereals

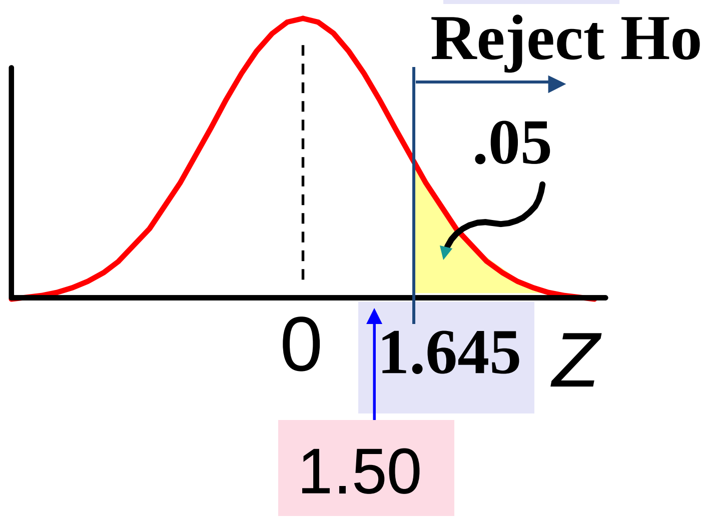
Example Solution: One Tail Test

Ho: $\mu = 368$ **grams**

Ha: $\mu >$ **368 grams**

α = **0.05;** ***n*** **= 25**

Critical Value: 1.645



Test Statistic:

$$Z_c = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}} = 1.5$$

Decision: Since $Z_c = 1.5 < 1.645$, we fail to Reject Ho at $\alpha = .05$

Conclusion:

The sample does not provide sufficient evidence to support that the true mean is more than 368.

Example: Two-Tail Test

Q. Does an average box of cereal contain 368 grams of cereal? A random sample of 25 boxes showed $\overline{X}$ = 372.5 grams. The company has specified σ to be 15 grams. Test at the $\alpha = 0.05$ level of significance.

$$Ho: \mu = 368 \text{ grams}$$

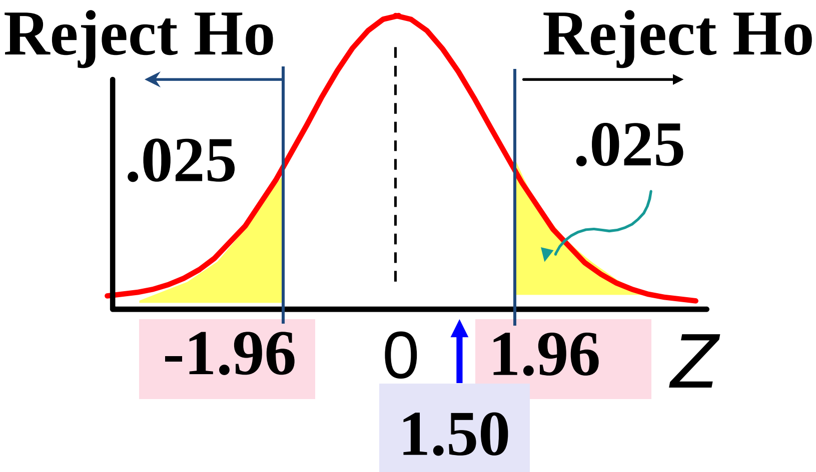
$$Ha: \mu \neq 368 \text{ grams}$$

Example Solution: Two-Tail Test

Ho: μ = 368 g.   ***Ha***: μ ≠ 368 g.

α = 0.05; n = 25

Critical Values: ±1.96



Test Statistic:

$$Z_c = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}} = \frac{372.5 - 368}{15 / \sqrt{25}} = 1.5$$

Decision: Since Zc = 1.5 < 1.96

We fail to Reject Ho at α = .05

Conclusion:

The sample does not provide sufficient evidence to support the claim that the mean is not equal to 368

Connection to Confidence Intervals

For $\bar{X} = 372.5$, $\sigma = 15$ and $n = 25$,

the 95% confidence interval is:

$$372.5 - (1.96)15/\sqrt{25} \le \mu \le 372.5 + (1.96)15/\sqrt{25}$$

or

$$366.62 \le \mu \le 378.38$$

For this example, the hypothesized value of the mean which is 368 falls in the interval estimate. Thus, we fail to reject Ho.

Example: One-Tail *t* Test

Does an average box of cereal contain more than 368 grams of cereal? A random sample of 36 boxes showed $\bar{X} = 372.5$ grams and $s = 15$ g. Test at the $\alpha = 0.01$ level of significance.

σ is not given

$H_0: \mu = 368$ **grams**

$Ha: \mu > 368$ **grams**

Example Solution: One-Tail

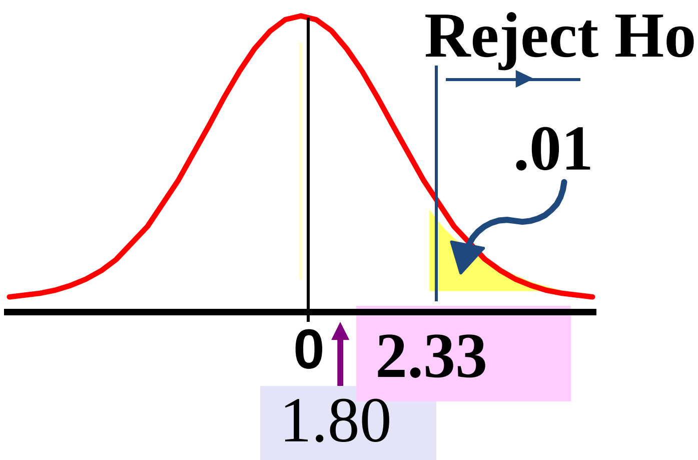
H_0: $\mu = 368$; Ha: $\mu > 368$

Test Statistic:

$$Z_C = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3725 - 368}{15/\sqrt{36}} = 1.80$$

α = 0.01; n = 36, df = 35

Critical Value: 2.33



Decision: Since $Z_C = 1.8 < 2.33$,

We fail to reject Ho at α = .01

Conclusion:

The sample does not provide sufficient evidence to support the claim that the true mean is more than 368.

Example:

Cooking oils that are low in both cholesterol and saturated fats are often recommended for people who are trying to lower their blood cholesterol level or to lose weight. Many cooking oils that have no cholesterol still have saturated fat contents of 6% to 18%. Cooking oil made from soybeans has been advertised as containing 15% saturated fats. A dietitian thinks that the percentage of saturated fats is greater than 15% and randomly selects 13 bottles of soybean cooking oil for testing. These bottles contain the following percentages of saturated fats:

15.2 12.4 15.4 13.5 15.9 17.1 20.0

16.9 14.3 19.1 18.2 15.5 16.3

On the basis of this sample, can the dietitian conclude that the level of saturated fats in cooking oil made from soybeans is greater than 15% at 0.01 level of significance? (Assume that the population is normally distributed.)

What would happen if instead of taking a sample of size 13, the dietitian takes a sample of size 39? Include the following additional observations in the original data set and test the same hypotheses at 0.01 level of significance.

15.2	12.4	15	13.5	15.9	17.1	20.0
16.9	14.3	19.1	18.2	15.5	16.3	
15.2	12.4	15.4	13.5	15.9	17.1	20.0
16.9	14.3	19.1	18.2	15.5	16.3	

PhStat Output

t Test for Hypothesis of the Mean

Data	
Null Hypothesis μ=	15
Level of Significance	0.01
Sample Size	13
Sample Mean	16.14
Sample Standard Deviation	2.154
Intermediate Calculations	
Standard Error of the Mean	0.597
Degrees of Freedom	12
t Test Statistic	1.906
Upper-Tail Test	
Upper Critical Value	2.681
p-Value	0.04
Fail to reject the null hypothesis	

Testing a Claim About a Proportion

We can test a claim about a proportion, percentage, or probability, as illustrated in these examples:

- Based on a sample survey, fewer than ¼ of all college graduates smoke.
- The percentage of physicians leaving the country exceeds 15%.
- If a driver is fatally injured in a car crash, there is a 0.35 probability that the driver was legally impaired.

Assumptions Used When Testing a Claim About a Population Proportion, Probability, or Percentage

1. The conditions for a binomial experiment are satisfied. That is, we have a fixed number of independent trials having constant probabilities, and each trial has two outcome categories, which we classify as "success" and "failure".
2. The conditions np $\geq$ 5 and nq $\geq$ 5 are both satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with μ = np and $\sigma = \sqrt{npq}$

Notation in Testing a Claim About a Proportion

Notation

n = number of trials

$\hat{p} = \frac{x}{n}$ (sample proportion)

p = population proportion (used in the null hypothesis)

q = 1-p

Testing a Claim About a Proportion

H_0: $p = p_o$

vs. Ha: $p > p_o$

Ha: $p < po$

Ha: $p \neq po$

Test Statistic:

$$Z_c = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1 - p_o)}{n}}}$$

Example: Z Test for Proportion

Q. 250 housewives were randomly selected and asked whether they prefer purchasing fish from supermarkets or from wet (public) markets.

If 114 of them preferred supermarkets, is there evidence at the 5% level of significance to suggest that the proportion of housewives throughout the city who prefer supermarkets exceeds 40%

$\hat{p}$

Example: Z Test for Proportion

- Check:

$$n \hat{p} = 250(0.4) = 100 \geq 5$$

$$n(1-\hat{p}) = 250(0.6) = 150 \geq 5$$

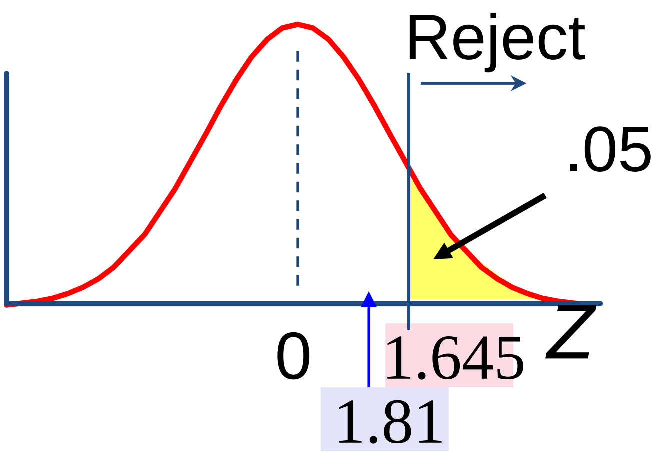
$\hat{p}$ is proportion of housewives in the sample who prefer supermarkets

Z Test for Proportion: Solution

H_0: $p = .4$ Ha: $p > .4$

α **= .05;** n **= 250**

Critical Values: 1.645



Test Statistic:

$$Z_c = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}} = \frac{\left(\frac{114}{250}\right) - 0.4}{\sqrt{\frac{0.4(1-0.4)}{250}}} = 1.81$$

Decision: Since Zc=1.81 < 1.96,

we fail to reject Ho at α = .05

Conclusion:

The sample do not provide sufficient evidence to support p > .4

Example for Testing the Claim About a Proportion

In a study of air-bag effectiveness, it was found that in 821 crashes of midsize cars equipped with air bags, 46 of the crashes resulted in hospitalization of the drivers.

Use a 0.01 level of significance to test the claim that the airbag hospitalization rate is lower than the 7.8% rate for crashes of midsize cars equipped with automatic safety belts.

Solution: Output from PhStat

Z Test of Hypothesis for the Proportion

Data	
Null Hypothesis ***p*=**	**0.078**
Level of Significance	**0.01**
Number of Successes	**46**
Sample Size	**821**
Intermediate Calculations	
Sample Proportion	0.056029233
Standard Error	0.009359253
Z Test Statistic	-2.347491574
Lower-Tail Test	
Lower Critical Value	**-2.326347**
***p*-Value**	**0.009450132**
Reject the null hypothesis	