MATH55 1st Long Exam

Directions: Show your complete solutions.

1. Given $f\left(x,y,z\right) = x^2y^3z^4$.
   a. Find the rate of change at the point $\left(1,-1,1\right)$ in the direction given by the vector $\vec{V} = \left\langle -2,1,2 \right\rangle$.
   b. What is the unit vector of the direction in which the rate of change of f at $\left(1,-1,1\right)$ is maximum?

2. Using Lagrange multipliers, find the maximum value of $f\left(x,y,z\right) = 2x + 6y + 10z$ satisfying $x^2 + y^2 + z^2 = 35$.

3. What is the equation of the tangent plane to the surface given by the vector equation $\vec{R}\left(s,t\right) = \left\langle s^2, s - t^2, t^3 \right\rangle$ at the point $\left(1,0,1\right)$ on the surface?

4. <u>*Setup*</u> the iterated integral for the volume of the solid under the paraboloid $z = 3x^2 + y^2$ over the region bounded by $y = -x$ and $y = x^2 - 6$.

5. <u>*Setup*</u> the iterated integral for the area of the surface given by $z = x + y^2$ over the triangular region with vertices at $\left(0,0\right)$, $\left(1,1\right)$ and $\left(0,1\right)$.

6. <u>*Evaluate*</u> the following iterated integrals.
   a. $\int_0^1 \int_{2y}^2 \cos x^2 \, dx dy$
   b. $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \left(x^2 + y^2\right)^{\frac{3}{2}} \, dx dy$

—End of Exam—