MATH55 **2^{nd} Long Exam**

Directions: Show your complete solutions.

1. Using spherical coordinates, evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2+y^2}} \frac{1}{x^2+y^2+z^2} dzdxdy$.

2. Setup an iterated integral in cylindrical coordinates that gives the volume of the solid in the first octant bounded by the cylinder $x^2+y^2=1$ and the plane $z=x$.

3. Given the vector field $\vec{F}(x,y,z)=\langle z-x, 2y, 2xz\rangle$
   a. Determine $div\ \vec{F}$ and $curl\ \vec{F}$.
   b. Find the work done by $\vec{F}$ in moving an object along the curve C defined by $\vec{R}(x,y,z)=\langle t^2, e^t, t-1\rangle,\ t\in[0,1]$.

4. Given the vector field $\vec{F}(x,y)=\left\langle \frac{y}{x}+\sin y, \ln x + x\cos y - 4y\right\rangle$
   a. Show that $\vec{F}$ is conservative.
   b. Find all potential functions of $\vec{F}$.
   c. Evaluate $\int_C \vec{F}\cdot d\vec{R}$, where C is any path from the point $(1,\pi)$ to $(e,0)$.

5. Use Green's Theorem to evaluate the line integral $\oint_C (x^2+y^2)dx + (x^2y)dy$, where C is the closed curve determined by $x=y^2$ and $y=-x$.

6. Evaluate the surface integral $\iint_S (2x-3y+z)dS$, where S is the surface given by the vector equation $\vec{R}(s,t)=\langle 2t, t-s, s\rangle$, where $0\le s\le 1$ and $0\le t\le 1-s$.

7. Find the flux of $\vec{F}(x,y,z)=\langle -1,0,1\rangle$ across the portion of the cone $z=\sqrt{x^2+y^2}$ which is inside the cylinder $x^2+y^2=1$ with upward orientation.