MATH55 **3rd Long Exam**

Directions: Show your complete solutions.

1. (a) Show that the sequence $\left\{\dfrac{2n^3 - 3}{5n^3 - 2n}\right\}_{n=1}^{\infty}$ is convergent.

   (b) Explain why the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3 - 3}{5n^3 - 2n}$ is divergent.

2. Find the sum of the series $\displaystyle\sum_{n=2}^{\infty}\left[\frac{2n-2}{n+1} - \frac{2n}{n+2}\right]$.

3. Determine whether the given series is convergent of divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^4 n}{n^3 + 5n + 2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 3n^3}{1 + 2n^4}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

4. Determine the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n \sqrt{n+1}}$

5. (a) Find a power series representation for $\dfrac{1}{2-x}$

   (b) Find a power series representation for $\dfrac{1}{(2-x)^2}$. (Hint: $\dfrac{1}{(2-x)^2} = D_x\left[\dfrac{1}{2-x}\right]$).

   (c) Use (b) to find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1} n}{2^{n+1}}$.

6. Let $f(x) = \sqrt{x}$.

   (a) Find the third degree Taylor polynomial of $f(x)$ about $x = 1$,

   (b) Using (a), estimate the numerical value of $\sqrt{1.01}$.

—End of Exam—