

I. TRUE or FALSE. Write **TRUE** if the statement is correct, and write **FALSE** otherwise. **Explain briefly.**

1. All sets are disjoint with the empty set.
2. $\frac{x+y}{x+z} = \frac{y}{z}$.
3. Given three non-empty sets A , B , and C , if $A \neq B$ and $B \neq C$ then $A \neq C$.
4. The additive inverse of the multiplicative inverse of $-\frac{2}{3}$ is $\frac{3}{2}$.
5. The number 3.1415926536 is a rational number.
6. $A \subseteq A, B$.
7. Division is an associative operation on the set of \mathbb{R} of real numbers.
8. For every $a \in \mathbb{R}$, there exists an inverse element for multiplication, $\frac{1}{a}$, such that $a \cdot \frac{1}{a} = 1$.
9. A nonnegative number is always positive.
10. Every integer is a factor of zero.
11. $c - (a - b) = c - a - b$
12. The set of even numbers is closed under addition.
13. The set of irrational numbers is closed under division.
14. The sum of two irrational numbers is always irrational.
15. $3(xy) = (3x)(3y)$ by distributive axiom.

II. Do as indicated.

1. List all the elements of the following:
 - (a) The set of even prime numbers.
 - (b) $\{b|b \text{ is a nonpositive integer greater than } 3\}$.
 - (c) $\{n|n \text{ is a positive divisor of } 9\}$.
 - (d) $\left\{ \frac{1}{x} \mid x \text{ is a positive integer less than } 10 \right\}$
2. Use the rule method to describe the sets listed below.
 - (a) $\{\text{Tuesday, Thursday}\}$
 - (b) $\{+, \times, -, \div, \sqrt{\quad}, ()^n\}$
 - (c) $\{1, 11, 111, 1111, \dots\}$
 - (d) $\{X, L, D, I, C, M, V\}$
3. Find the *gcd* and the *lcm* of a and b .
 - (a) $a = 8, b = 28$
 - (b) $a = 27, b = 63$
 - (c) $a = 2^2 \cdot 3^3 \cdot 5, b = 2^3 \cdot 3^3 \cdot 7$
 - (d) $a = 1, b = p$

4. The power set of a set A or the set of subsets of A is defined to be $\wp(A) = \{X | X \subseteq A\}$.
 - (a) Give the set of subsets (power set) of $X = \{1, \{2, 3\}\}$.
 - (b) If $\wp(A)$ has only one element, what can we say about A ?
 - (c) What is the cardinality of $\wp(A)$ if $n(A) = 1?, 2?, 3?, n?$
 - (d) If $X = \{a, b\}$, then what is $\wp(\wp(X))$
5. A survey of the reading habits of 90 residents of a community showed 40 read newspaper X, 30 read newspaper Y, 20 read newspaper Z, 15 read newspapers X and Y, 10 read newspapers X and Z, 9 read newspapers Y and Z and 7 read all three. How many read
 - (a) At least one newspaper? at most one?
 - (b) Newspaper X but not Y?
 - (c) none of these newspapers?
6. Fill in the blanks using the following symbols: $\mathbb{N}, W, \mathbb{Z}, \mathbb{Q}, \mathbb{Q}', \mathbb{R}, \emptyset, \cup, \cap, \setminus$.
 - (a) $\mathbb{Q}' \cap W = \underline{\hspace{2cm}}$
 - (b) $\mathbb{R} \underline{\hspace{0.5cm}} \mathbb{Q} = \mathbb{Q}$
 - (c) $(\mathbb{N} \cup W) \cap \mathbb{Z} = \underline{\hspace{2cm}}$
 - (d) $(\mathbb{R} \setminus \mathbb{Z}) \setminus \mathbb{Q} = \underline{\hspace{2cm}}$
 - (e) $\mathbb{R} \setminus (\mathbb{N} \underline{\hspace{0.5cm}} \mathbb{Q}) = \mathbb{Q}'$

III. Give the axiom that justifies each statement below.

1. $3 \cdot 1 = 1 \cdot 3$
2. $7 \cdot 1 = 7 \cdot 1$
3. $\sqrt{3} \cdot \pi$ is a real number.
4. If $a + b = c + d$ and $c + d = 0$, then $a + b = 0$.
5. If a is neither positive nor negative, then $a = 0$.
6. If $x > 0$ and $y < 0$, then $x > y$.

IV. Evaluate and simplify the following.

1. $(2^{-3})^{-1}$
2. $(2^{-1} + 3^{-2})^{-1}$
3. $\frac{3^0 + 3^{-1}}{3^{-3} + 3^0}$
4. $\frac{(20r^2s^3t^4)(2r^2s^2t)}{(-4rst)(3rs^4t^2)}$
5. $\left(\frac{7x^5}{8w^3y^4z^6}\right)^5$
6. $\left(\frac{5x^2y^{-4}}{4x^{-2}y^{-3}}\right)^3$
7. $\left(\frac{3x^3y}{2x^{-2}y^3}\right)^4 \left(\frac{3x^{-4}y^2}{4xy^{-2}}\right)^{-3}$

*Examples from CAT by Castillo, et. al, CAT by Leithold, MAT3rd by Vance
Also, courtesy of manjologs*