

I. Do as indicated

1. Evaluate the following:

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|---|---|
| (a) $\cos 165^\circ$ | (i) $\frac{2 \tan 7.5^\circ}{1 - \tan^2 7.5^\circ}$ |
| (b) $\tan\left(\frac{13\pi}{12}\right)$ | (j) $\sin 255^\circ + \sin 345^\circ$ |
| (c) $\cot 112.5^\circ$ | (k) $\cos\left(\frac{13\pi}{8}\right) \sin\left(\frac{7\pi}{8}\right)$ |
| (d) $\sec\left(\frac{23\pi}{8}\right)$ | (l) $\sin \frac{13\pi}{20} \cos \frac{7\pi}{20} + \cos \frac{7\pi}{5} \sin \frac{11\pi}{10}$ |
| (e) $\csc 75^\circ$ | (m) $\frac{4 \sin^4\left(\frac{\pi}{12}\right) - 1}{2 \sin^2\left(\frac{\pi}{12}\right) + 1}$ |
| (f) $\sin\left(\frac{19\pi}{8}\right)$ | (n) $\sin^2 0^\circ + \sin^2 1^\circ + \dots + \sin^2 89^\circ + \sin^2 90^\circ$ |
| (g) $\cos 50^\circ \sin 275^\circ + \sin 50^\circ \cos 275^\circ$ | |
| (h) $\sin 110^\circ \sin 70^\circ - \cos 110^\circ \cos 70^\circ$ | |

2. Solve for each of the following completely.

- (a) Find $\tan \frac{\alpha}{2}$ and $\cos 2\alpha$ if $\sin \alpha = \frac{12}{13}$ and α is not in **I**.
- (b) Find the exact values of $\sin(\alpha + \beta)$ and $\tan(\alpha - \beta)$ if $\cos \alpha = -\frac{4}{5}$, $\sin \beta = \frac{5}{13}$ and α and β are in the second quadrant.
- (c) Find the exact values of $\cot(\alpha + \beta)$ and $\sec(\alpha - \beta)$ if $\tan \alpha = \frac{3}{4}$, $\cos \beta = \frac{4}{5}$ and α and β are in the first quadrant.
- (d) If $\sin 2\theta = \frac{1}{4}$, what is the value of $(\cos \theta + \sin \theta)^6$.

II. Prove the following identities.

- $\frac{\csc x}{\csc x - \sin x} = \sec^2 x$
- $\frac{\sin s}{\csc s - \cot s} = 1 + \cos s$
- $\frac{\sec s + \tan s}{\csc s} = \sec s + \tan s - \cos s$
- $\frac{1}{\cos s} - \frac{\cos s}{1 + \sin s} = \tan s$
- $\csc 2\theta + \cot 2\theta = \cot \theta$
- $\cot \beta = \frac{1 + \sin 2\beta + \cos 2\beta}{1 + \sin 2\beta - \cos 2\beta}$
- $\frac{\sin x - \sin y}{\sin x + \sin y} = \frac{\tan \frac{x-y}{2}}{\tan \frac{x+y}{2}}$
- $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta = 4 \cos \theta \cos 2\theta \cos 3\theta$
- $\frac{\sin 7A - \sin 5A}{\cos 7A + \cos 5A} = \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$

*Examples from CAT by Castillo, CAT by Leithold
Courtesy of manjologs*