

I. TRUE or FALSE. Write **TRUE** if the statement is correct, and write **FALSE** otherwise.

1. The set of imaginary numbers is closed under addition and multiplication.
2. For all  $a \in \mathbb{R}, m, n \in \mathbb{N}, \sqrt[n]{\sqrt[m]{n}} = \sqrt[mn]{a}$
3. For all  $a \in \mathbb{R}, m, n \in \mathbb{N}, a^{\frac{m}{n}} = \sqrt[n]{a^m}$

II. Simplify the following.

1. Rational Exponents and Radicals

- (a)  $(-64)^{-\frac{2}{3}}$
- (b)  $\left[ \frac{25a^{\frac{4}{3}}b^{\frac{1}{2}}}{4a^{\frac{1}{3}}b^{\frac{2}{3}}} \right]^{-\frac{3}{2}}$
- (c)  $\left[ \frac{8^{\frac{2}{3}} - 4x^{-2}}{x^{-1} - x^0} \right]^{\frac{1}{2}}$
- (d)  $b^{-\frac{2}{3}} \left( b^{-\frac{1}{3}} - b^{\frac{2}{3}} \right)$
- (e)  $\left( a^{\frac{1}{4}} - a^{\frac{1}{2}} \right) \left( a^{-\frac{1}{4}} - a^{-\frac{1}{2}} \right)$
- (f)  $\frac{a^2 - 2a^{-1} - a^{-\frac{1}{2}}}{a^{\frac{1}{2}}}$
- (g)  $\frac{(x+1)^{\frac{1}{2}} - \frac{1}{2}x(x+1)^{-\frac{1}{2}}}{\left[ (x+1)^{\frac{1}{2}} \right]^2}$

2. Radicals

- (a)  $\sqrt[6]{(400)(324)}$
- (b)  $\sqrt[3]{4x^2y^2} \sqrt[3]{6x^2z^2} \sqrt[3]{45x^2y^2z}$
- (c)  $\frac{270u^2vw^3}{\sqrt[4]{288u^5v^6w^2}}$
- (d)  $\sqrt{\frac{1}{5}} - 3\sqrt{\frac{1}{125}} + \sqrt{20}$
- (e)  $\sqrt[4]{36} - \sqrt{54} + \sqrt{96}$
- (f)  $\frac{5\sqrt{3} - 4\sqrt{5}}{2\sqrt{5} + 3\sqrt{3}}$
- (g)  $\frac{9}{\sqrt[4]{9}} + \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}$
- (h)  $\frac{1}{\sqrt[3]{4} - 2}$

III. Do the following.

1. Express in standard form.

- (a)  $(4 - 2\sqrt{-45}) - (3 + \sqrt{63})$
- (b)  $\sqrt{-12}\sqrt{-16}\sqrt{-27}$
- (c)  $\frac{2 - 3i}{1 + 5i}$
- (d)  $(i - 1)^2 - (-i - 1)^2 + i^4$
- (e)  $i + i^2 + i^3 + \dots + i^{2010}$
- (f)  $\left( \left( \left( (i^1)^2 \right)^3 \right) \dots \right)^{2010}$

2. If  $z = a + bi$ , we define the *modulo* of  $z$  by  $|z| = \sqrt{z \cdot \bar{z}}$ . Find:

- (a)  $|4 + 3i|$
- (b)  $|2 - i|$
- (c)  $|i^2|$

*Examples from CAT by Castillo, et. al, CAT by Leithold  
Also, courtesy of manjologs*