

I. Polynomial Functions.

1. Which among the following: 1, -2, 2, -1, 3 is a zero of the function

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24.$$

2. Show that 3 is a zero of multiplicity two of the polynomial function defined by

$$P(x) = x^4 - 3x^3 - 11x^2 + 39x - 18$$

and find the other two zeros.

3. Prove that the equation $x^3 - 9x - 6 = 0$ has no rational roots.
 4. If $-2 + i$ is a zero of $P(x) = x^3 + 2x^2 - 3x - 10$, what are the other zeros?
 5. Find the remainder of $f(x) = x^4 + x^3 - 31x^2 - x + 30$ when it is divided by $(x + 4)$, $(x - 3)$, $(x + 1)$ and $(x - 2)$.
 6. Find all the rational zeros of the following:
 (a) $P(x) = x^3 - x^2 - 8x + 12$
 (b) $P(x) = 3x^3 + 8x^2 - 1$
 (c) $P(x) = 2x^3 - 7x^2 + 2x + 6$
 (d) $P(x) = 4x^5 - 18x^4 + 24x^3 - 7x^2 - 4x + 4$
 7. Find a non-zero degree equation who has roots 1, -1 and $-2i$ whose coefficients are integers.
 8. Using Descartes' rule of signs, determine the number of positive and negative roots of the following:
 (a) $4x^4 - 9x^2 + 12x^3 - 47x - 30 = 0$
 (b) $2x^5 + 3x^4 + 15x^2 - 32x + 12 = 0$
 (c) $5x^2 - 6x + 3x^3 - 10 = 0$
 9. Show that $\sqrt{3}$ is irrational using polynomials.
 10. Find the values of k so that $x + 1$ is a factor of $5x^3 + k^2x^2 + 2kx - 3$

II. Inverse Functions.

1. Find dom f , ran f and f^{-1} (if it exists). If the inverse does not exist, determine the largest set where the function has an inverse.
 (a) $f(x) = \sqrt{x - 4}$
 (b) $f(x) = \frac{x}{x - 3}$
 (c) $f(x) = (x - 2)^2$
 (d) $f(x) = 5x^3 + 2$
 (e) $f(x) = \sqrt[3]{\frac{x - 2}{5}}$
2. Determine if each function is one to one. If not, explain.
 (a) $\{(2, 3), (3, 5), (5, 7)\}$
 (b) $\{(x, y) | y = |x + 2|\}$
3. Find the domain, range and the inverse of the function
 $f(x) = \frac{2e^x + 1}{e^x - 1}$.

III. Exponential and Logarithmic Functions.

1. Sketch the graph of the following:
 (a) $y = -3^x$
 (b) $y = 3^{1-x} - 2$
2. Given $\log 5 = 0.699$ and $\log 3 = 0.4771$, evaluate $\log \sqrt[3]{\frac{24}{25}}$.
3. Solve for the following equations for x .
 (a) $4^{4x-3} = 8^{2x+5}$
 (b) $\log_3(x + 1) + \log_3(x + 3) = 1$
 (c) $3^{x^2} = (9^x)^2$
 (d) $\log_2(\log_5 x) = 1$
 (e) $2 \log_3(x + 3) - \log_3(x + 1) = 3 \log_9 4$
 (f) $\log_a \sqrt[3]{x^2} + \log_a \sqrt[4]{x^3} = \log_a 2^{-3}$
 (g) $27^{x+1} - 9^{\frac{2x+3}{2}} + 3^{x+2} = 1$

*Examples from CAT by Castillo, et. al, CAT by Leithold,
 Precalculus by Barnett, et. al, MAT3rd by Vance*