

I. Variations.

1. If a function $f(x)$ varies inversely as the cube of x , find the value of $\frac{f(2)}{f(3)}$.
2. The volume of a cone varies jointly as the square of the radius of the base and the height. If the radius of the base of the cone is 2cm. and it's height is 3cm., the volume of the cone is $4\pi\text{cm}^3$. If we want the height of the cone to be 27 cm without changing the volume, what should be the radius of the base of the cone?
3. If p varies directly as q and inversely as the square root of r , what is the effect on p if q is doubled and r is quadrupled?

II. Sequences and Series.

1. If the sequence 2, 6, 10, ... form an AP, which term is 106?
2. Evaluate $\sum_{k=0}^4 k(k-2)$
3. Find the value of x such that $2x+1, x-2$ and $3x+4$ form an arithmetic sequence.
4. The sum of the first three numbers of an arithmetic progression is 42. If the product of the first number and the second number in the progression is 140, find the sum of the first twenty terms of the progression.
5. Show that if $n \in \mathbb{N}, n \neq 1$, then $\frac{1}{\log_2 5} + \frac{1}{\log_3 5} + \frac{1}{\log_4 5} + \dots + \frac{1}{\log_n 5} = \log_5 n!$ where $n! = 1 \cdot 2 \cdot \dots \cdot n$.
6. Let $f(x) = 29 + 26x + 23x^2 + \dots + t_k x^k$ be a polynomial function. Determine the value of k so that the remainder when $f(x)$ is divided by $(x-1)$ is 10.
7. Show that $x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \dots + x^2 + x + 1)$.
8. Simplify $2^{2009} - 2^{2008} - 2^{2007} - \dots - 4 - 2 - 1$.
9. Show that if u, v , and w form a geometric progression then $\log u, \log v$ and $\log w$ form an arithmetic progression.
10. A *harmonic progression* is a sequence of numbers whose reciprocals form an arithmetic progression. Insert two harmonic means between 4 and 8.

III. Supplementary Exercises

1. Evaluate $(\log_2 3)(\log_3 4)(\log_4 5) \cdot \dots (\log_{2009} 2010)$.
2. If $c = \log 196$ and $d = \log 56$, find $\log(0.175)^4$.
3. If $a = \log_{12} 18$ and $b = \log_{24} 52$, find $ab + 5(a-b)$.