Exercises: Review of Functions

A. Find the domain of the following functions.

1. $f(x) = \dfrac{2x+1}{x^2-1}$
2. $f(x) = \sqrt{\dfrac{4}{2x-1} - \dfrac{1}{x+1}}$
3. $f(x) = \sqrt{x^2-3x+2} + \dfrac{3}{\sqrt{16-x^2}}$
4. $g(x) = \dfrac{x+1}{\sqrt{|x+1|-9}}$
5. $g(t) = \sqrt{8-t} - \sqrt[3]{t}$

B. Graph the following functions.

1. $f(x) = \dfrac{2x^2-2}{x+1}$
2. $g(x) = \sqrt{x-2}$
3. $h(x) = \sqrt{4-x^2}$
4. $m(x) = 1 - \sqrt{2 - \frac{1}{2}x}$
5. $p(x) = 2 - \sqrt{4-(x-2)^2}$

C. Express the following as a piecewise function and sketch the graph of the function.

1. $f(x) = \operatorname{sgn}(x^2-2x-3)$
2. $g(x) = -|x+2| - 5$
3. $h(x) = |x-2| - x$
4. $m(x) = |x^2-2x-8|$
5. $p(x) = [\![-x]\!]$
6. $F(x) = [\![2x-1]\!]$

D. Graph the following piecewise functions.

1. $f(x) = \begin{cases} 2-|x| & \text{if } x \le 3 \\ \dfrac{x^2-4x}{x-4} & \text{if } 3 < x < 5 \\ 2 & \text{if } x > 5 \end{cases}$
2. $f(x) = \begin{cases} 1 & \text{if } x > 4 \\ [\![x+1]\!] & \text{if } 2 \le x \le 4 \\ -x^2+7 & \text{if } x < 2 \end{cases}$
3. $f(x) = \begin{cases} -x^2+4x-3 & \text{if } 1 \le x < 4 \\ x\,[\![x]\!] - 1 & \text{if } -2 \le x < 1 \\ \dfrac{x^2-10x+24}{2x-12} & \text{if } x \ge 4 \end{cases}$

E. Express the following as a composition of functions discussed in class.

1. $F(x) = (x^2+1)^{10}$
2. $G(\theta) = \cos^4 2\theta$
3. $H(t) = \dfrac{\tan t}{1+\tan t}$
4. $f(x) = \tan\left(\sqrt{\dfrac{2x-1}{x+1}}\right)$

F. Find $f+g$, $f-g$, $f \cdot g$, f/g, $f \circ g$ and $g \circ f$.

1. $f(x) = 2x^2+1$; $g(x) = \sqrt{x}$
2. $f(x) = |1-x|$; $g(x) = \sqrt{\dfrac{2}{x} - 1}$

G. Let $g(x) = x+h$, where h is a very small, positive real number. Find $\dfrac{1}{h}\left[(f \circ g)(x) - f(x)\right]$.

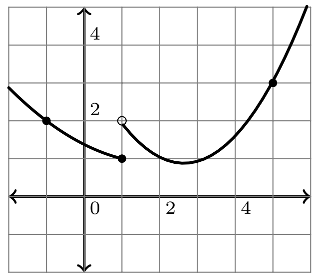
1. $f(x) = x^2+2x+1$
2. $f(x) = \dfrac{2}{x}$
3. $f(x) = \dfrac{x+1}{x-1}$
4. $f(x) = \sin x$

H. Mathematical Modeling.

1. Express the slope of the line joining $(1,0)$ and any point on the graph of the semicircle below the x-axis, centered at the origin with radius of one, as a function of x.
2. A closed tin can of volume 8π cubic meters is to be in the form of a right circular cylinder. If the material of the top and bottom cost Php1.00 per square meter and the material of the side costs Php2.00 per square meter express the cost of the manufacture of the cylinder as a function of its radius.
3. A conical cup has a radius of 2 inches across the top and a height of 9 inches. If the cup contains a liquid, express the volume of the liquid inside the cup as a function of height of the liquid.

Exercises: Limits

A. Let f be the function whose graph is shown in the figure below:



Determine $f(-1)$, $f(1)$, and $f(5)$. Determine also $\lim_{x\to -1} f(x)$, $\lim_{x\to 1} f(x)$ and $\lim_{x\to 5} f(x)$.

B. Evaluate the following limits.

1. $\lim_{w\to 1} (1+\sqrt[3]{w})(2-w^2+3w^3)$
2. $\lim_{s\to -1} (s^2+1)^3\sqrt{s^6+s^2+8}$
3. $\lim_{t\to -2} \dfrac{t^2-1}{t^2+3t+1}$
4. $\lim_{z\to 2} \left(\dfrac{2z-z^2}{z^2-4}\right)^3$
5. $\lim_{x\to -1} \dfrac{x^2-x-2}{x^3-6x^2-7x}$
6. $\lim_{y\to -2} \dfrac{4-3y^2-y^3}{6-y-2y^2}$
7. $\lim_{x\to 4} \dfrac{x^3-7x^2+14x-8}{x^2-3x-4}$
8. $\lim_{x\to -1} \dfrac{\sqrt{x^2+3}-2}{x^2-1}$
9. $\lim_{x\to 2} \dfrac{\sqrt{2x}-\sqrt{6-x}}{4-x^2}$
10. $\lim_{t\to 3} \dfrac{\sqrt{7-t}-2}{\sqrt{4-t}-1} + \dfrac{6}{t}$
11. $\lim_{u\to 1} \dfrac{\sqrt[3]{u+7}-2}{1-u}$
12. $\lim_{p\to 1} \dfrac{p^3-1}{\sqrt{2p-1}-1}$

13. $\displaystyle\lim_{x\to 1}\frac{\sqrt{2x+2}-2}{\sqrt[3]{x}-1}$

14. $\displaystyle\lim_{q\to -1}\frac{\sqrt{9q^2-4}-\sqrt{17+12q}}{q^2+3q+2}$

C. Let $f(x)=\dfrac{\sin x}{x}$. Using a calculator, compute for $f(x)$ when $x=\pm 0.1, \pm 0.001, \pm 0.000001$. Based on your results, what could the value of $\displaystyle\lim_{x\to 0} f(x)$ be?

D. Let $f(x)=\dfrac{1-\cos x}{x}$. Using a calculator, compute for $f(x)$ when $x=\pm 0.1, \pm 0.001, \pm 0.000001$. Based on your results, what could the value of $\displaystyle\lim_{x\to 0} f(x)$ be?

Exercises: One-sided limits

A. Evaluate the following limits.

1. $\displaystyle\lim_{x\to -2^+}\sqrt{x^2-4}$
2. $\displaystyle\lim_{x\to -2^-}\sqrt{8-x+x^3}$
3. $\displaystyle\lim_{x\to 1^+}\sqrt{\frac{2-x-x^2}{x+3}}$
4. $\displaystyle\lim_{x\to 2^-}\left(\frac{1}{x}-\frac{1}{[\![x]\!]}\right)$
5. $\displaystyle\lim_{x\to -2^+}\left(\frac{1}{[\![-x]\!]}-\frac{1}{|x|}\right)$
6. $\displaystyle\lim_{t\to 4}(2t+|t-4|)$
7. $\displaystyle\lim_{x\to 0}\frac{|3x+2|-|x-2|}{x}$
8. $\displaystyle\lim_{x\to 1}\frac{|4x|-|x-5|}{1-x}$
9. $\displaystyle\lim_{x\to 2^+}\frac{[\![x]\!]-1}{[\![x]\!]-|x|}$
10. $\displaystyle\lim_{x\to 1^-}\frac{x[\![x]\!]-1}{[\![x]\!]-1}$
11. $\displaystyle\lim_{y\to 4^+}\frac{[\![2-y]\!]}{[\![y+2]\!]-y}$
12. $\displaystyle\lim_{x\to 2}\frac{x^2-[\![x+2]\!]x}{x-[\![2x]\!]}$

B. Let

$$g(x)=\begin{cases} x, & \text{if } x<0\\ 3, & \text{if } x=0\\ 3-x^2, & \text{if } 1<x\le 4,\\ x-3 & \text{if } x>4\end{cases}$$

Evaluate:

1. $\displaystyle\lim_{x\to 0^-} g(x)$
2. $\displaystyle\lim_{x\to 0^+} g(x)$
3. $\displaystyle\lim_{x\to 0} g(x)$
4. $\displaystyle\lim_{x\to 1^-} g(x)$
5. $\displaystyle\lim_{x\to 1^+} g(x)$
6. $\displaystyle\lim_{x\to 4} g(x)$

C. Let

$$f(x)=\begin{cases} \dfrac{x^2-4}{x+2}, & x\le 0\\ x^2+2x-3, & 0<x<2\\ x+3, & x\ge 2\end{cases}$$

Evaluate:

1. $\displaystyle\lim_{x\to 0} f(x)$
2. $\displaystyle\lim_{x\to 2} f(x)$
3. $\displaystyle\lim_{x\to -2} f(x)$

D. Let

$$g(x)=\begin{cases} \dfrac{x^2-4}{x^2+2x}, & x<-1\\ 3, & x=-1\\ \dfrac{x+1}{\sqrt{2x+3}-1}, & x>-1\end{cases}$$

Evaluate:

1. $\displaystyle\lim_{x\to -2} g(x)$
2. $\displaystyle\lim_{x\to -1} g(x)$
3. $\displaystyle\lim_{x\to 0} g(x)$

E. Given

$$h(x)=\begin{cases} kx-3, & x\le -1\\ x^2+k, & x>-1\end{cases}$$

where k is constant. Find k so that $\displaystyle\lim_{x\to -1} h(x)$ exists.

F. Let

$$f(x)=\begin{cases} a-3x & x<-2\\ ax+3b & -2\le x\le 1\\ 4x+b & x>1\end{cases}$$

Find a and b so that $\displaystyle\lim_{x\to -2} f(x)$ and $\displaystyle\lim_{x\to 1} f(x)$ both exist.

G. 1. Let $f(x)=\begin{cases} \dfrac{x^2+4x+3}{x^2-x-2}, & x\le -1\\ [\![x+3]\!], & -1<x<1\\ \sqrt{x-1}, & x\ge 1\end{cases}$

Find $\displaystyle\lim_{x\to -1} f(x)$, $\displaystyle\lim_{x\to 0} f(x)$, $\displaystyle\lim_{x\to 1} f(x)$, $\displaystyle\lim_{x\to 2} f(x)$.

2. Let $g(x)=\begin{cases} \dfrac{|2x-x^2|}{x}, & x<0\\ \dfrac{[\![x+3]\!]-6}{3-[\![x]\!]}, & 0\le x<3\\ \dfrac{x^2-6x+5}{x^3-19x-30}, & x\ge 3\end{cases}$

Find $\displaystyle\lim_{x\to 0} g(x)$, $\displaystyle\lim_{x\to 3} g(x)$, $\displaystyle\lim_{x\to 5} g(x)$.

H. Sketch a graph of a function $f(x)$ satisfying the following:

- $\operatorname{dom} f=[-4,4]$
- $f(-4)=f(-2)=3$
- $f(0)=1$
- $f(2)=-1$
- $f(4)=0$
- $\displaystyle\lim_{x\to -4^+} f(x)=0$
- $\displaystyle\lim_{x\to -2} f(x)=1$
- $\displaystyle\lim_{x\to 0^-} f(x)=1$
- $\displaystyle\lim_{x\to 0^+} f(x)=4$
- $\displaystyle\lim_{x\to 2} f(x)=-1$
- $\displaystyle\lim_{x\to 4^-} f(x)=0$

Exercises: Infinite limits

Compute the following limits.

1. $\displaystyle\lim_{s\to -2}\frac{2-|s|}{2+s}$
2. $\displaystyle\lim_{t\to 0}\left(\frac{1}{t\sqrt{t+3}}-\frac{1}{t}\right)$
3. $\displaystyle\lim_{x\to 3^+}\frac{2x^2-7x+3}{|9-x^2|}$
4. $\displaystyle\lim_{s\to -1}\left(\frac{2}{s^2-1}+\frac{1}{s^2+3s+2}+\frac{7s}{s^3+8}\right)$
5. $\displaystyle\lim_{x\to 2^+}\frac{[\![x]\!]-1}{[\![x]\!]-x}$
6. $\displaystyle\lim_{x\to 2^-}\frac{[\![x]\!]-1}{[\![x]\!]-x}$
7. $\displaystyle\lim_{x\to 1^+}\frac{[\![x^2]\!]-[\![x]\!]^2}{x^2-1}$