UP SCHOOL OF STATISTICS STUDENT COUNCIL
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M53_LE4_003
Mathematics 53 — Elementary Analysis I
Fourth Long Examination — Second Semester AY 2015-2016

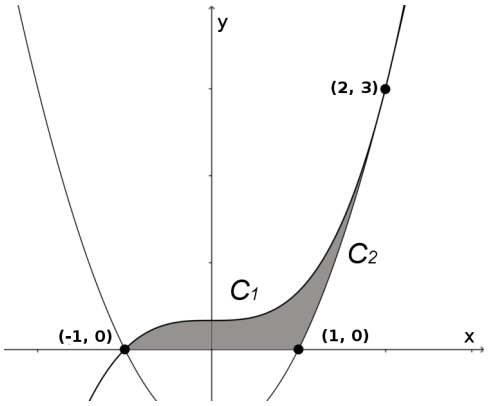
This is an 80-minute exam. Except for Part V, provide neat, complete and organized solutions, and box your final answers. Use black or blue non-erasable ink only. Any form of cheating in examinations shall be subject to disciplinary action.

I. Evaluate the following integrals. *(4 pts each)*

1. $\int 4x(2x-5)^{19}\,dx$
2. $\int_0^{\pi/4} (\tan x - \cos x)\tan x\,dx$
3. $\int_1^4 \frac{24x}{3x+6+|3x-6|}\,dx$
4. $\int \frac{\cos(\sqrt[3]{x})\sin^2(\sqrt[3]{x})}{\sqrt[3]{x^2}}\,dx$

II. Let R be the shaded region bounded by the graphs of $C_1 : y = \frac{x^3+1}{3}$, $C_2 : y = x^2 - 1$, and the x-axis as shown below. **SET UP** the definite integral(s) equal to:

1. The area of R. *(3 pts)*
2. The volume of the solid formed when R is revolved about the line $x = 3$
   (a) using washers. *(3 pts)*
   (b) using cylindrical shells. *(3 pts)*
3. The arc length of the graph of $y = \frac{x^3+1}{3}$ from the point $(-1,0)$ to the point $(2,3)$. *(2 pts)*



III. Suppose that a particle on the origin travels along a straight path with an initial velocity of 4 m/s, and for any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 2$.

1. Find the position function $s(t)$ of the particle at any time t. *(3 pts)*
2. How far is the particle from the origin at time $t = 1$? *(1 pt)*

IV. Suppose $F(x) = \int_{3x}^{\frac{3\pi}{2}} \sin^5 t\,dt$, where x is any real number.

1. Evaluate $F(\frac{\pi}{2})$. (*1 pt*)
2. Find $F'(x)$. (*3 pts*)

V. Write TRUE if the statement is correct, and FALSE otherwise. (*1 pt each*)

1. The average value of $y = \cos x$ over the interval $[a,b]$ is equal to the value of the slope of the line joining the points $(a, \sin a)$ and $(b, \sin b)$.
2. If $f(x)$ is a function and c is in the domain of f, then $\int_a^b f(c)\,dx = f(c)(b-a)$.
3. If f is continuous on $[a,b]$ and $\int_a^b f(x)\,dx = 0$, then f is *only* either increasing on $[a,b]$ or decreasing on $[a,b]$.
4. An antiderivative of $f(x) = -\csc x \cot^2 x - \csc^3 x$ is $g(x) = \csc x \cot x$.
5. If n is a positive integer and $0 < \mathrm{k} < 1$, then $\int_n^{n+k} [\![x]\!]\,dx = nk$.

END OF EXAM
Total: 40 points