Mathematics 54 **Fourth Long Exam**

General Direction: *Use black or blue ballpen. Show neat and complete solution to obtain full points.*

I. Do as indicated.

1. Let $\vec{R}(t) = \left\langle \dfrac{1-\cos 3t}{t}, t\ln t, \dfrac{t-\sin t}{e^t-1} \right\rangle$. Evaluate $\lim_{t\to 0^+} \vec{R}(t)$. **3 points**

2. Consider the curve $C : x = 1+\cos t,\ y = \sin^2 t$.
   a. Determine the *curvature* and *radius of curvature* of C at $t = \frac{\pi}{2}$. **4 points**
   (**HINT:** Take the third component to be zero.)
   b. Find the cartesian equation of the curve C. **1 point**

3. Let $\vec{R}(t) = e^t\cos t\hat{i} + e^t \sin t\hat{j} + \sqrt{2}e^t\hat{k}$.
   Find the *arclength parametrization* of $\vec{R}$ given the initial point at $t = \ln 3$. **4 points**

4. The velocity function of a moving particle is given as $\vec{V} =< e^t, e^{-t}, \frac{1}{2} >$. Assume that the particle is initially at the origin.
   a. Find the position and acceleration at any time t. **3 points**
   b. Determine the *moving trihedral* of the vector-valued function $\vec{R}(t)$ at $t = \ln 2$. **5 points**
   c. The *osculating plane* of the *Frenet frame* is produced by the unit normal and unit tangent vectors. Find the equation of the normal plane at $t = \ln 2$. **1 point**
   d. Find the *scalar* tangential and normal component of acceleration, and *vector* tangential and normal componenet of acceleration. **4 points**

MORE AT THE BACK

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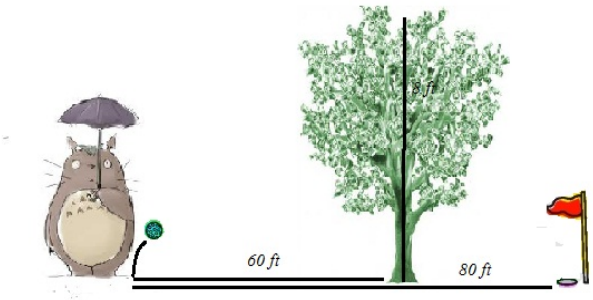
MORE AT THE BACK

5. A tree $8ft$ tall, stands directly between the pin and the golf ball which is $80ft$ from the pin. The tree is $60ft$ from the ball. A golfer hits the ball with speed of $48ft/s$ at an angle of $\theta = \frac{\pi}{4}$.
   a. Find the position function of the ball. **3 points**
   b. Show that the golf ball didn't hit the tree. **3 points**
   c. At what time did the ball reach the ground? **2 points**
   d. After landing, how many feet is the ball away from the pin? **2 points**



END OF EXAM
TOTAL: 35 POINTS

"Before you met me, I was a wreck; but things were kinda heavy, you brought me to life."
- Katy Perry (Teenage Dreams)

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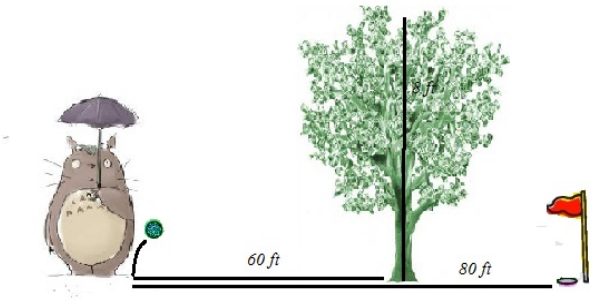

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