

Mathematics 54 First Long Exam M54\_LE1\_002 Elementary Analysis II First Semester, AY 2014-2015

I. Solve the following indefinite integrals.

1. 
$$\int \sin x (\cos(5x) + \sin^2 x) dx$$
  
2.  $\int \frac{dx}{(x^2 - 9)^{3/2}}$   
3.  $\int \frac{4x^2 - 3x + 7}{(x - 2)(4x^2 + 1)} dx$   
4.  $\int \sin(2x) e^{\cos x} dx$ 

II. Evaluate the following improper integrals, if they converge.

- 1.  $\int_{-2}^{1} \frac{x^2}{\sqrt{x^3 + 8}} dx$  2.  $\int_{-\infty}^{+\infty} \frac{x^3}{1 + x^4} dx$
- III. Find the equation of the curve passing through  $(0, \frac{3\pi}{4})$  if the slope of the tangent line at any point (x, y) on the curve equals  $\cos^2 y \sin^2 x$ . (5 points)

IV. Find the orthogonal trajectories of  $y = \frac{k}{1+x^2}$ , where  $k \in \mathbb{R}$ . (5 points)

## Total: 40 points

(5, 6, 6, 5 points)

(4 points each)