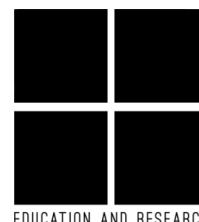


# UP SCHOOL OF STATISTICS STUDENT COUNCIL

## Education and Research

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### Mathematics 54 Second Long Exam

M54\_LE2\_001  
Elementary Analysis II  
1st Semester AY 2016-2017

This is an 80-minute exam. Except for Part VI, provide neat, complete and organized solutions, and box your final answers. Use black or blue non-erasable ink only. Any form of cheating in examinations shall be subject to disciplinary action.

I. Given the conic with equation  $4(y - 4)^2 - 9x^2 = 36$ . (6 points)

1. Identify the center, vertices, foci and endpoints of the conjugate axis.
2. Sketch the graph of the conic. Label important points with their coordinates.

II. Given a curve with parametric equations  $x = \ln(t - 1) + (t - 1)^{-1}$ ,  $y = \ln(t - 1)$  where  $t > 1$ .

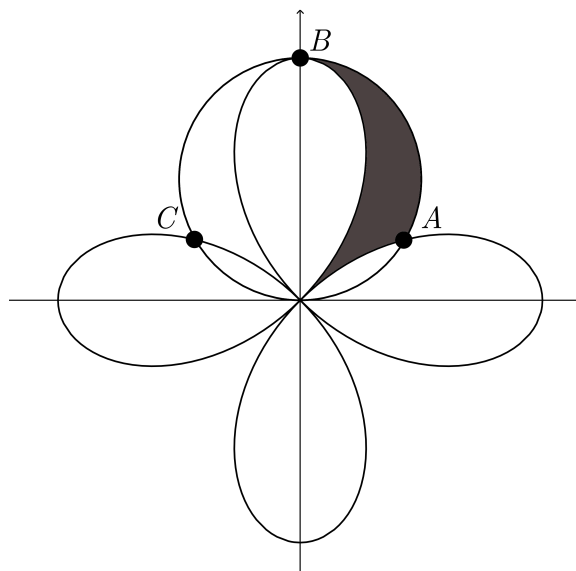
1. Find the values of  $t$  where the curve has horizontal and vertical tangent lines. (3 points)
2. Evaluate  $\left. \frac{d^2y}{dx^2} \right|_{t=2}$  without eliminating the parameter  $t$ . (2 points)
3. SET UP the integral (in terms of  $t$ ) equal to the arc length of the curve from the point where  $t = 2$  to the point  $(1 + e^{-1}, 1)$ . (3 points)
4. Find a Cartesian equation for the curve of the form  $x = f(y)$ . (2 points)

III. Find a Cartesian equation of the tangent line to the curve  $r = 5 + 4 \sin \theta$  at the point where  $\theta = \frac{\pi}{6}$ . (5 points)

IV. Find a polar equation, in the form  $r = f(\theta)$ , of the conic with eccentricity  $\frac{3}{4}$ , a focus at  $(0, 0)$  and corresponding directrix  $x = -\frac{5}{3}$ . (2 points)

V. Let  $C_1 : r = \sin \theta$  and  $C_2 : r = \cos 2\theta$  as shown in the figure.

1. Find the polar coordinates  $(r, \theta)$  of the points  $A$ ,  $B$  and  $C$ . (3 points)
2. Determine the subinterval of  $\theta \in [0, 2\pi]$  that will trace the petal containing the point  $B$ . (1 point)
3. Using the formulas for arclength and area of polar curves, SET UP the integral(s) equal to the following: (3 points each)
  - (a) the area of the shaded region
  - (b) the length of the arc traced counterclockwise along the curve  $C_1$  from point  $C$  to point  $A$



VI. Write the CAPITAL letter of the correct answer.

(1 point each)

1. Which of the following curves is symmetric with respect to the polar axis?

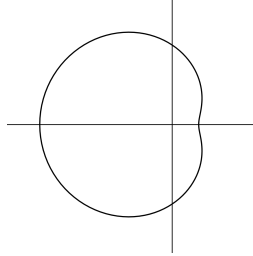
A)  $\theta = \sin 2$

B)  $r = 2 \sin \theta$

C)  $r = 2 \sin 2\theta$

D)  $r = 2 + 2 \sin \theta$

2. Which of the following is a polar equation for the graph of the limaçon below?



A)  $3 + 2 \cos \theta$

B)  $3 - 2 \cos \theta$

C)  $5 + 2 \cos \theta$

D)  $5 - 2 \cos \theta$

3. What is the graph of the equation  $r = \frac{2}{\cos \theta}$ ?

A) horizontal line

B) vertical line

C) parabola

D) circle

4. Which of the following is satisfied by the conic with equation  $16x^2 + 9y^2 = 144$ ?

A)  $a = 3, b = 4, c = 5$

C)  $a = 3, b = 4, c = \sqrt{7}$

B)  $a = 4, b = 3, c = 5$

D)  $a = 4, b = 3, c = \sqrt{7}$

5. Let  $E_1$  and  $E_2$  be the endpoints of the latus rectum of the parabola  $y^2 = 4\left(\frac{1}{2}\right)x$ , and let  $P$  be the point on the directrix nearest  $E_1$ . What is the distance between  $E_2$  and  $P$ ?

A)  $\sqrt{2}$

B)  $\sqrt{3}$

C)  $\sqrt{4}$

D)  $\sqrt{5}$

For items 6 and 7:

The vertices of a conic are 2 units apart while its foci are 3 units apart. Suppose that a point  $P$  on the conic is  $\frac{1}{2}$  unit away from the nearest directrix.

6. What is the eccentricity of the conic?

A)  $\frac{2}{3}$

B)  $\frac{3}{2}$

C)  $\frac{3}{4}$

D)  $\frac{4}{3}$

7. How far is the point  $P$  from the nearest focus?

A)  $\frac{2}{3}$

B)  $\frac{3}{2}$

C)  $\frac{3}{4}$

D)  $\frac{4}{3}$

\*\*\* END OF EXAM \*\*\*

Total: 40 points