M54_LE2_005
Mathematics 54 — Elementary Analysis II
Second Long Exam — First Semester, AY 2013-2014

Show all necessary solutions. Write legibly and box every final answer. Use black or blue pen only.

I. Multiple choice (5 pts)

1. The graph of $r = 5 - 2\sin\theta$ is a:

   a. limaçon with a loop
   b. cardioid
   c. limaçon with a dent
   d. convex limaçon

2. The graph of $r = \dfrac{2}{4 - \cos\theta}$ is:

   a. a parabola
   b. an ellipse
   c. a hyperbola
   d. none of the above

3. The rose $r = 4\cos 3\theta$ is symmetric with respect to:

   a. the polar axis
   b. the $\frac{\pi}{2}$-axis
   c. both a and b
   d. none of the above

4. The eccentricity of the ellipse $\dfrac{x^2}{64} + \dfrac{y^2}{100} = 1$ is:

   a. $\frac{3}{4}$ b. $\frac{3}{5}$ c. $\frac{\sqrt{41}}{4}$ d. $\frac{\sqrt{41}}{4}$

5. The parabola $y^2 = 4 - 4x$ has directrix:

   a. $x = 1$ b. $y = 1$ c. $x = 2$ d. $x = -1$

II. Find the length of the polar curve $r = e^{-2\theta}$, $\theta \in [0, \ln 2]$. (4 pts)

III.

1. Write the Cartesian equation in standard form of the hyperbola

$$r = \frac{3}{1 + 2\cos\theta}.$$ (4 pts)

2. Give the Cartesian coordinates of its foci and vertices. (3 pts)

IV.

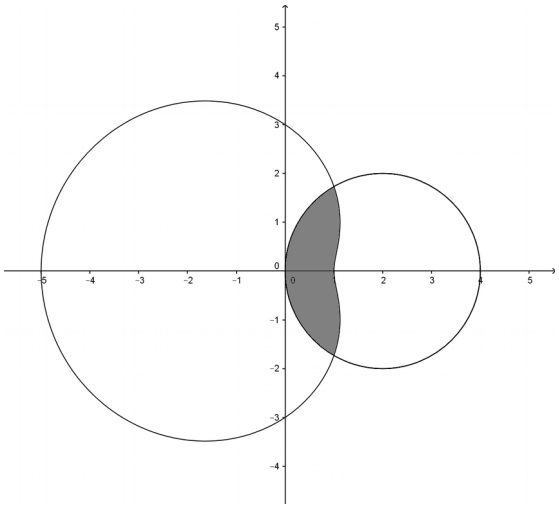
1. Find the points on the cardioid $r = 1 + \cos\theta$ where the tangent line is horizontal. (4 pts)

2. Given the parametric equations $x = \ln t$ and $y = t^3 + 1$ where $t > 0$, find $\dfrac{d^2y}{dx^2}$ without eliminating t. (2 pts)

V.

1. Give a set of parametric equations for the line segment from (1, 2) to (−1, 0). (2 pts)

2. Sketch the curve defined by the parametric equations $x = \cos t$ and $y = e^{\cos t}$. (2 pts)



VI. Refer to the figure above.

1. Find the points of intersection of the limaçon $r = 3 - 2\cos\theta$ and the circle $r = 4\cos\theta$. (3 pts)

2. **Set up** the integral which gives the area of the region inside both the limaçon and the circle. (4 pts)

VII.

1. Find the equation of any of the two parabolas whose vertex is the center of the ellipse

$$\frac{(x-2)^2}{25} + \frac{(y-1)^2}{16} = 1$$

and whose focus is also a focus of the given ellipse. (3 pts)

2. In the same coordinate plane, draw both the ellipse and the parabola. Label the foci and vertices of both conics. (4 pts)

End
Total: 40 points

Any form of cheating in examinations or any act of dishonesty in relation to studies, such as plagiarism shall be subject to disciplinary action.