UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

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EDUCATION AND RESEARCH

M54_LE3_001

MATHEMATICS 54 — **Elementary Analysis II**

Third Long Exam — **1st Semester AY 2016-2017**

INSTRUCTIONS: Write your answers clearly and legibly. Show ALL necessary solutions and BOX all final answers. Use black or blue pen only. This exam is for 80 minutes only.

1. Let S be the quadric surface $x^2 + \dfrac{y^2}{9} = 1 - z$.
   (a) Identify the given quadric surface. *(1 point)*
   (b) Give the equations and identify the traces of S on the coordinate planes. *(3 points)*
   (c) Sketch S. Label its intercepts and the traces obtained in (b). *(3 points)*

2. Consider the points $A(2,\,0,\,1)$ and $C(-6,\,6,\,3)$.
   (a) Find the midpoint B of the line segment joining A and C. *(2 points)*
   (b) Give an equation of the sphere centered at C and that passes through B. *(3 points)*

3. Give the equation of the surface generated when the curve $z = \ln(x+3)$ is revolved about the $z-$axis. *(2 points)*

4. Do as indicated.
   (a) Let $\vec{v} = \langle 8\sqrt{5},\, 2\sqrt{5},\, \sqrt{15}\rangle$ and $\vec{w} = \left\langle -\frac{1}{\sqrt{5}},\, \frac{1}{\sqrt{5}},\, \frac{\sqrt{3}}{\sqrt{5}} \right\rangle$. Find the magnitude of $proj_{\vec{w}}\vec{v}$. *(3 points)*
   (b) Use scalar triple product to find the volume of the parallelipiped with $\vec{v} = \langle 0,\,-2,\,3\rangle$, $\vec{w} = \langle 3,\,1,\,3\rangle$, and $\vec{v}\times\vec{w}$ as adjacent edges. *(3 points)*

5. Let $\ell_1 : x + 2 = 5 - y = z - 2$ and $\ell_2 : \begin{cases} x = -3 + 2t \\ y = 4 \\ z = 6 - 3t \end{cases}$
   (a) Find the point of intersection of ℓ_1 and ℓ_2. *(4 points)*
   (b) Find the equation of the plane π containing both ℓ_1 and ℓ_2. *(3 points)*
   (c) Find the distance between plane π and the point $(0,\,6,\,1)$. *(2 points)*

6. Determine whether the following statements are true or false. Write TRUE if the given statement is true. Otherwise, write FALSE. Show all necessary solutions to justify your answer. *(2 points each)*
   (a) Consider the vectors $\vec{u} = \langle -18,\,-7,\,17\rangle$, $\vec{v} = \langle 4,\,3,\,-3\rangle$, and $\vec{u} = \langle -3,\,1,\,4\rangle$. Then there is a nonzero vector $\vec{x}$ satisfying the equation
   $$\vec{u} + 3\vec{v} - 2(\vec{x} + \vec{w}) = \vec{0}$$
   .
   (b) If $\vec{v}$ and $\vec{w}$ are nonzero vectors in 3-space that are perpendicular to each other then $\dfrac{\vec{v}\times\vec{w}}{\|\vec{v}\|\|\vec{w}\|}$ must be a unit vector.

(c) There is a vector of length 6 such that the value of its direction cosines are $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{2}$.

***** END OF EXAM *****
Total: 35 points