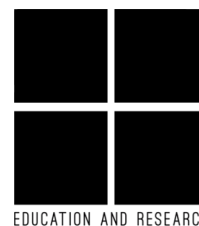


# UP SCHOOL OF STATISTICS STUDENT COUNCIL

## Education and Research

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### Mathematics 54 Third Long Exam

M54\_LE3\_002  
Elementary Analysis II  
1st Semester AY 2016-2017

This exam is for 80 minutes only. Use black or blue non-erasable ink only. Show neat and complete solutions, and box all final answers. The use of electronic devices is not allowed during the exam.

**Any form of cheating in examinations or any act of dishonesty in relation to studies, such as plagiarism, shall be subject to disciplinary action.**

I. Write TRUE if the statement is always true. Otherwise, write FALSE. [1 pt each]

- The projection of  $\vec{a}$  onto  $\vec{b}$  is parallel to  $\vec{a}$ .
- The graph of the equation  $z = x^2 + y^2$  is a hyperbolic paraboloid.
- If two lines  $\ell_1$  and  $\ell_2$  do not intersect, then  $\ell_1$  is parallel to  $\ell_2$ .
- If  $\vec{a} + \vec{b} = \vec{0}$  and  $\vec{a} - \vec{b} = \vec{0}$ , then  $\vec{a} = \vec{b} = \vec{0}$ .
- If  $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$ , then  $\vec{u} \times \vec{v}$  is orthogonal to  $\vec{w}$ .

II. Given the vectors

$$\vec{u} = \langle -1, 1, 2 \rangle, \vec{v} = \langle 6, 0, 3 \rangle, \vec{w} = \langle 0, 3, -4 \rangle$$

- Show that  $\vec{u}$  and  $\vec{v}$  are orthogonal. [2 pts]
- Find the vector of length 3 with direction opposite to the direction of  $\vec{w}$ . [2 pts]
- Find the volume of the parallelepiped defined by the vectors  $\vec{u}$ ,  $\vec{v}$ , and  $\vec{w}$ . [3 pts]

III. Given a point  $P(2, 1, -1)$  two lines  $\ell_1 : \begin{cases} x = 3 + t \\ y = 2 + t \\ z = -6 - 2t \end{cases}$  and  $\ell_2 : \frac{x+2}{3} = y+1 = \frac{z-2}{-4}$ .

- Find the equation (in center-radius form) of the sphere centered at  $P$  and passing through the point on  $\ell_1$  with  $x$ -coordinate equal to 0 [3 pts]
- Find the point of intersection of  $\ell_1$  and  $\ell_2$ . [3 pts]
- Find the distance between  $P$  and  $\ell_1$ . [4 pts]

IV. Consider the planes  $\pi_1 : 2x - y + 3z + 5 = 0$  and  $\pi_2 : x + 2y - 2z - 4 = 0$ , and the point  $Q(0, 7, -4)$ .

- Find an equation of the plane passing through  $Q$  and perpendicular to  $\pi_1$  and  $\pi_2$ . [3 pts]
- Find symmetric equations of the line passing through  $Q$  and perpendicular to  $\pi_1$ . [2 pts]
- Find the distance between  $Q$  and  $\pi_2$ . [2 pts]

V. Consider the equation  $z = x^2$

- Sketch the portion of the cylinder with the given equation in the first octant. [2 pts]
- Let  $C$  be the curve in the  $xz$ -plane with the given equation. Find an equation of the surface of revolution obtained by revolving  $C$  about the  $z$ -axis. [2 pts]

VI. Given the quadric surface with equation  $4 - z = \frac{x^2}{4} + \frac{y^2}{9}$ .

1. Find the equation of the traces on the  $xy$ -,  $xz$ -, and  $yz$ -planes. Identify these traces. *[3 pts]*
2. Identify the surface. *[1 pt]*
3. Sketch the graph of the surface. Label important points on the graph. *[3 pts]*

**END OF EXAM**  
**TOTAL: 40 points**