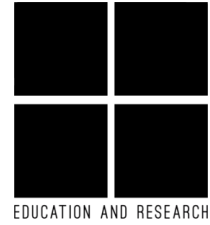




UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

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Mathematics 54 4th Long Exam

M54_LE4_001
Elementary Analysis II
1st Semester, A.Y. 2016-2017

Instruction. This exam is good for 80 minutes only. Show neat and complete solutions on each problem to gain full points. Use only blue or black non-erasable ink. DO NOT TEAR OFF any page from your bluebook. The use of electronic devices is not allowed during the exam. BOX all final answers.

1. Find all values of $a \in \mathbb{R}$ such that:

$$\vec{R}(t) = \begin{cases} \left\langle t^3 - at + a^2, t^2 + 1, \frac{\sin(\pi t)}{at - 4} \right\rangle & , t \neq 2 \\ \langle 8, 5, \pi/2 \rangle & , t = 2 \end{cases}$$

is continuous at $t = 2$

[4 pts]

2. Let \vec{R} be a vector valued function such that $\vec{R}(1) = \langle 3, 1, 1 \rangle$, $\vec{R}'(1) = \langle 2, 2, -1 \rangle$ and $\vec{R}''(1) = \langle 0, 2, 2 \rangle$. Find:

(a) the vector function for the tangent line to the graph of \vec{R} at $t = 1$.

[2 pts]

(b) $\vec{Q}'(1)$ where $\vec{Q}(t) = \cos(\pi t)\vec{R}'(t)$

[3 pts]

(c) the equation of the osculating plane at $t = 1$.

[3 pts]

(d) the radius of curvature when $t = 1$.

[3 pts]

3. Find a vector equation for the curve of intersection of the elliptic paraboloids $5x^2 + y^2 = 16 - z$ and $z = 3x^2 + 3y^2$.

[3 pts]

4. Eevee, in search of her mother Sylveon, is walking along a curve C given by:

$$\vec{R}(t) = (2e^t - 1)\hat{i} + (2e^t + 1)\hat{j} + e^t\hat{k}$$

if Eevee started walking from the point $P(1, 3, 1)$,

(a) find the arclength parametrization of $\vec{R}(t)$ with P as its reference point.

[4 pts]

(b) If Eevee needs to walk 6 units along C from P to find Sylveon, find the coordinates of Sylveon's location.
[2 pts]

5. A golden retriever puppy is running along a path where its velocity is given by

$$\vec{V}(t) = \left\langle 2e^{2t} - 4, 3t^2 - 1, \frac{2}{t+1} \right\rangle.$$

Initially, the puppy is located at $(1, 1, 0)$.

(a) Determine the acceleration and position vector of the puppy for any time t .

[5 pts]

(b) Find the normal and tangential components of the puppy's acceleration at $t = 0$.

[4 pts]

6. From the top of a cliff, a cannon ball is fired at an angle 30° with initial speed 70 m/s. The cannon ball landed to the ground $350\sqrt{3}$ meters away from the base of the cliff. Assuming that the acceleration due to gravity is $g = 10 \text{ m/s}^2$, find the following:

- (a) the time when the cannon ball hits the ground; [2 pts]
- (b) the height of the cliff. [2 pts]
- (c) the time when the cannon ball reaches its maximum height; [3 pts]

END OF EXAM

Total: 40 points

"Any form of cheating in examinations or any act of dishonesty in relation to studies, such as plagiarism, shall be subject to disciplinary action."