UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research
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EDUCATION AND RESEARCH

M54_LE4_002

Mathematics 54 | **Elementary Analysis II**
Fourth Long Exam | **1st Semester AY 2016-2017**

General Directions: The exam is to be taken within 80 minutes. Provide what is asked for in each item. Show complete solutions and box your final numerical answers. Use black and blue pen only. NO PENCILS. Any form of cheating will not be tolerated.

Any form of cheating in examinations or any act of dishonesty in relation to studies, such as plagiarism, shall be subject to disciplinary action.

1. Find the domain of $\vec{R}(t) = \left\langle \frac{3t-3}{t^2-1}, \ln(4-t^2), e^{-t} \right\rangle$. *(2 points)*

2. Is $\vec{R}(t) = \begin{cases} \left\langle \frac{\sin(t-3)}{t-3}, \sqrt{t^2+3}, \cos^{-1}\left(\frac{t}{6}\right) \right\rangle & , \text{if } t \neq 3 \\ \hat{\imath} + 2\sqrt{3}\hat{\jmath} + \frac{\pi}{6}\hat{k} & , \text{if } t = 3 \end{cases}$ continuous at $t = 3$? *(4 points)*

3. Find parametric equations for the curve of intersection of the surface $S_1 : z = 3x^2+5y^2$ and $S_2 : z = y^2-x^2+4$. *(3 points)*

4. Given $\vec{R}(t) = \langle t - \cos t, t^2 - \sin t + 1, \tan^{-1} t \rangle$, $\vec{Q}(0) = \langle 1, 0, -2 \rangle$, $\vec{Q}'(0) = \langle 0, 3, 2 \rangle$:
   (a) Find the vector equation of the tangent line to the graph of $\vec{Q}$ at $t = 0$. *(2 points)*
   (b) Find the vectors $\vec{R}(0)$, $\vec{R}'(0)$, and $\vec{R}''(0)$. *(4 points)*
   (c) Evaluate $\frac{d}{dt}\left(\vec{R}'(t) \cdot \vec{Q}(t)\right)$ at $t = 0$. *(3 points)*

5. Given $\vec{R}(0) = \langle 2, 0, -3 \rangle$, $\vec{R}'(0) = \langle 0, -1, 1 \rangle$, $\vec{R}''(0) = \langle 3, -2, 0 \rangle$:
   (a) Find the equation of the osculating plane to the graph of $\vec{R}$ at the point when $t = 0$. *(3 points)*
   (b) Find the radius of curvature of $\vec{R}$ at $t = 0$. *(2 points)*

6. Reparametrize $\vec{R}(t) = \left\langle 4t, (4-t)^{\frac{3}{2}}, t^{\frac{3}{2}} \right\rangle$ using the arc length parameter about the point $(0, 8, 0)$. *(4 points)*

7. A particle is moving in space with velocity $\vec{V}(t) = (2\cos t)\hat{\imath} + (2\tan t)\hat{\jmath} - (2\sin t)\hat{k}$ such that when $t = 0$, it is at the point $(0, 2, 1)$. Find
   (a) the position vector $\vec{R}$ of the particle at any time t. *(3 points)*
   (b) the distance travelled by the particle from $t = 0$ to $t = \pi/4$. *(2 points)*
   (c) the normal and tangential components of acceleration at $t = \pi/4$. *(4 points)*

8. A projectile is fired from the ground at an angle of elevation θ with an initial speed of 40 m/s. Its range is 80 m. Assume that the acceleration due to gravity is 10 m/s^2. Find the value of the angle of elevation θ. *(4 points)*

Total: 40 points