UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research
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EDUCATION AND RESEARCH

Mathematics 55
Final Exam

M55_FIN_001
Elementary Analysis III
1st Semester AY 2014-2015

I. Let $f(x,y) = x^2 + y^2 + x^2y - 1$.

1. Find the rate of change of f at the point $P(2,1)$ along $\vec{v} = \langle 1, -1\rangle$. (3pts)
2. Find all relative extrema and saddle points of f. (4pts)

II. **Set up** a triple integral in **rectangular coordinates** that gives the volume of the solid enclosed by the plane $x - 2y + z = 2$ and the three coordinate planes. (4pts)

III. Use **spherical coordinates** to find the mass of the solid in the first octant bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 4$ if the density at any point in the solid is $\delta(x,y,z) = \sqrt{x^2 + y^2 + z^2}$ (5pts)

IV. Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{R}$ where $\vec{F}(x,y,z) = \left\langle x + y, z, x^2y \right\rangle$ and C is given by the vector function $\vec{R}(t) = \left\langle 2t, t^2, t^4 \right\rangle,\ t \in [0,1]$. (5pts)

V. Given $\vec{F}(x,y) = \left\langle 2xy\cos\left(x^2\right), \sin\left(x^2\right) - 6y \right\rangle$:

1. Show that $\vec{F}$ is conservative. (2pts)
2. Find a potential function for $\vec{F}$. (3pts)
3. Use the Fundamental Theorem for Line Integrals to evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{R}$ if C is any smooth arc from $(0,1)$ to $\left(\sqrt{\frac{\pi}{2}}, 2\right)$. (2pts)

VI. Use Green's Theorem to evaluate $\displaystyle\oint_C \left(\cosh\left(x^2+1\right) - 2y\right) dx + \left(3x + \tan^{-1}\left(y^2 - 1\right)\right) dy$ where C is the circle defined by the vector function $\vec{R}(t) = \langle 2\cos t, 2\sin t\rangle,\ t \in [0, 2\pi]$ (4pts)

VII. Compute the flux of $\vec{F}(x,y,z) = \langle x, y, z\rangle$ across the positively oriented portion of the paraboloid $z = 4 - x^2 - y^2$ above the $xy-$plane. (5pts)

VIII. Determine the convergence or divergence of each of the following.

1. $\displaystyle\left\{\frac{\ln n}{n}\right\}_{n=2}^{\infty}$ (2pts)
2. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$ (3pts)

IX. Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{2n+1}$.

1. Find its radius of convergence. (3pts)
2. Determine the values of x for which the series converges. (4pts)

X. Given $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ x \in \mathbb{R}$

1. Determine the Maclaurin series for xe^{x^2}. (3pts)

2. Hence, find the sum $\sum_{n=0}^{\infty} \frac{1}{2^{2n+1} n!}$. (3pts)

END OF EXAM
Total: 55 points