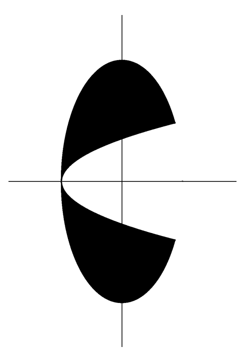
M55_LE1_001
Mathematics 55 **Elementary Analysis III**
First Long Exam **2nd Semester AY 2014-2015**

1. Let $f(x,y) = \dfrac{x^2}{2} - y^4$.
   (a) Determine the directional derivative of f at $(-2,1)$ along $\langle 3,-4\rangle$.
   (b) Give a direction vector from the point $\left(3,\frac{1}{2}\right)$ where the following happens:
      i. f decreases most rapidly
      ii. the rate of change of f is zero
2. Suppose $f(x,y)$ is differentiable on $\mathbb{R}^2$ with $f_x(x,y) = x^2 + 2y$ and $f_y(x,y) = 2x - y$. At each critical point, use the Second Derivative Test to determine whether there is a relative minimum, relative maximum or saddle point.
3. Use the method of Lagrange multipliers to find the absolute extreme values of $f(x,y) = 1 - y^2x$ subject to the constraint $x^2 + y^2 - 2y = 1$.
4. Let S be the surface defined parametrically by $x = v\cos w,\ y = w\cos w,\ z = v + w$ where $v,w \in \mathbb{R}$. Find the equation of the tangent plane S at the point $(v,w) = (0,\pi)$.
5. Evaluate the double integral $\displaystyle\int_0^1 \int_{2y}^2 \sec^2\left(x^2\right) dxdy$.
6. **Set-up** an iterated double integral in polar coordinates equivalent to the double integral
$$\int_{-2}^0 \int_2^{2+\sqrt{4-x^2}} \frac{x}{x^2+y^2}\, dydx.$$
7. Find the surface area of the portion of the hemisphere $z = \sqrt{5 - x^2 - y^2}$ between the planes $z = 1$ and $z = 2$.
8. Consider a lamina in the shape of the region bounded by $y^2 = 16 - 4x^2$ and $y^2 = x + 2$ (see figure below). **Set-up** the iterated integral(s) equal to the mass of the lamina if its density at each point (x,y) is given by $\delta(x,y) = 4 + y$.



END OF EXAM