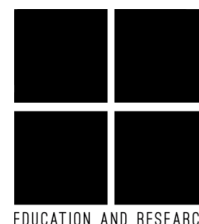


# UP SCHOOL OF STATISTICS STUDENT COUNCIL

## Education and Research

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Mathematics 55  
First Long Exam

M55\_LE1\_005  
Elementary Analysis III  
2nd Semester AY 2013-2014

**Directions:** Write all answers clearly and legibly in blue or black ink. You have 90 minutes to finish this exam.

- Let  $f(x, y) = y^2 e^{2x} + \cos(xy)$  and  $P_0(0, 2)$ . [6 pts.]
  - Find the directional derivative of  $f$  at the point  $P_0$  in the direction of  $\vec{v} = \langle 1, -1 \rangle$ .
  - Find the maximum rate of change of  $f$  at  $P_0$ .
  - Find a unit vector in the direction in which  $f$  decreases most rapidly at  $P_0$ .

- Determine the equations of the normal line to the surface  $S$  at the point  $(0, 0, 1)$  if  $S$  is described by the Cartesian equation  $\ln(2y + z) = xz^2$ . [4 pts.]

- Let  $S$  be the parametric surface defined by [4 pts.]

$$\vec{R}(u, v) = uv^2\vec{i} + (u - v)\vec{j} + u^2\vec{k}.$$

- Compute  $\vec{R}_u \times \vec{R}_v$ .
  - Find an equation of the tangent plane to  $S$  at the point  $(1, 2, 1)$ .
- Find and classify all critical points  $(x, y)$  of the function  $f(x, y) = x^3 + 5y^3 - 3x^2y - 3y + 1$ . [6 pts.]
  - Use the method of Lagrange multipliers to find the maximum and minimum values of the function  $f(x, y, z) = 10x - 8y + 6z$  subject to the constraint  $x^2 + y^2 + z^2 = 50$ . [4 pts.]

- Evaluate  $\int_0^{6\sqrt{\pi}} \int_{y/6}^{\sqrt{\pi}} \sin(x^2) dx dy$ . [4 pts.]

- Evaluate by converting to polar coordinates:  $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{x^2 + y^2} dy dx$  [4 pts.]

- Set up** an iterated double integral that yields the (surface) area of the portion of the plane  $2x + 2y + z = 4$  **in the first octant** enclosed by the  $yz$ -plane and the cylinder  $y = x^2$ . [4 pts.]

- Set up** an iterated double integral in polar coordinates that gives the mass of a lamina in the shape of the region **in the first quadrant** inside the circle  $x^2 + y^2 = 9$  but outside the circle  $x^2 + (y - 1)^2 = 1$ , having density  $\delta(x, y) = x^3 y$ . [4 pts.]

**End of Exam**  
**Total: 40 Points**

"A student shall be subject to disciplinary action for any form of cheating in examinations or any act of dishonesty in relation to his studies." - Article 12.1 Section 2, UP Diliman Faculty Manual