UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research
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EDUCATION AND RESEARCH

M55_LE1_005
Mathematics 55 **Elementary Analysis III**
First Long Exam **2nd Semester AY 2013-2014**

Directions: *Write all answers clearly and legibly in blue or black ink. You have 90 minutes to finish this exam.*

1. Let $f(x,y) = y^2e^{2x} + \cos(xy)$ and $P_0(0,2)$. [6 pts.]
   (a) Find the directional derivative of f at the point P_0 in the direction of $\vec{v} = \langle 1, -1\rangle$.
   (b) Find the maximum rate of change of f at P_0.
   (c) Find a unit vector in the direction in which f decreases most rapidly at P_0.
2. Determine the equations of the normal line to the surface S at the point $(0,0,1)$ if S is described by the Cartesian equation $\ln(2y+z) = xz^2$. [4 pts.]
3. Let S be the parametric surface defined by [4 pts.]
$$\vec{\boldsymbol{R}}(u,v) = uv^2\vec{\boldsymbol{i}} + (u-v)\vec{\boldsymbol{j}} + u^2\vec{\boldsymbol{k}}.$$
   (a) Compute $\vec{\boldsymbol{R}}_u \times \vec{\boldsymbol{R}}_v$.
   (b) Find an equation of the tangent plane to S at the poinst $(1,2,1)$.
4. Find and classify all critical points (x,y) of the function $f(x,y) = x^3 + 5y^3 - 3x^2y - 3y + 1$. [6 pts.]
5. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x,y,z) = 10x - 8y + 6z$ subject to the constraint $x^2 + y^2 + z^2 = 50$. [4 pts.]
6. Evaluate $\displaystyle\int_0^{6\sqrt{\pi}} \int_{y/6}^{\sqrt{\pi}} \sin(x^2)\, dxdy$. [4 pts.]
7. Evaluate by converting to polar coordinates: $\displaystyle\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dydx$ [4 pts.]
8. **Set up** an iterated double integral that yields the (surface) area of the portion of the plane $2x + 2y + z = 4$ **in the first octant** enclosed by the yz-plane and the cylinder $y = x^2$. [4 pts.]
9. **Set up** an iterated double integral in polar coordinates that gives the mass of a lamina in the shape of the region **in the first quadrant** inside the circle $x^2 + y^2 = 9$ but outside the circle $x^2 + (y-1)^2 = 1$, having density $\delta(x,y) = x^3y$. [4 pts.]

End of Exam
Total: 40 Points

"A student shall be subject to disciplinary action for any form of cheating in examinations or any act of dishonesty in relation to his studies." - Article 12.1 Section 2, UP Diliman Faculty Manual