UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research
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EDUCATION AND RESEARCH

M55_LE2_001

Mathematics 55
Second Long Exam

Elementary Analysis III
2nd Semester AY 2014-2015

1. Let G be a solid in the first octant bounded below by the cone $z = \sqrt{\dfrac{x^2+y^2}{3}}$ and above the plane $z = 3$. Suppose that the density at a point (x, y, z) in G is $f(x, y, z) = 2z$.
   (a) Set up an iterated triple integral that gives the mass of G using rectangular coordinates and spherical coordinates.
   (b) Find the mass of G using spherical coordinates.
2. Let $\vec{F}(x, y, z) = \left\langle zye^{x^2}, xze^{y^2}, xye^{z^2} \right\rangle$. Find the divergence of $\vec{F}$ and curl of $\vec{F}$.
3. Let $\vec{F}(x, y, z) = \langle 2x\cos y, e^z - x^2 \sin y, ye^z \rangle$.
   (a) Show that $\vec{F}$ is conservative by finding a potential function for $\vec{F}$.
   (b) Find the work done by $\vec{F}$ on a particle that moves on any smooth curve from the point $(0, 1, 0)$ to the point $(2, 0, 3)$.
4. Evaluate $\displaystyle\int_C (xy + y^2)\, ds$ where C is the lower half of the circle $x^2 + y^2 = 9$, described in the counterclockwise direction.
5. Evaluate $\displaystyle\int_C y\, dx + z\, dy - x\, dz$ where C is the line segment from $(0, 1, 2)$ to $(1, 3, 6)$ followed by the line segment from $(1, 3, 6)$ to $(1, 3, 2)$.
6. Let $\vec{F}(x, y) = \langle 2xy, xy + x^2 \rangle$. Using Green's Theorem, evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{R}$, where C is the triangular path traced in the counterclockwise direction with vertices at the points $(0, 0)$, $(1, 0)$ and $(2, 1)$.
7. Find the flux of $\vec{F}(x, y, z) = \langle x, y, z \rangle$ across the part of the positively-oriented paraboloid $z = 1 - 4x^2 - 4y^2$ above the xy-plane.

END OF EXAM