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EDUCATION AND RESEARCH

M55_LE2_005

Mathematics 55 **Elementary Analysis III**
Second Long Exam **2nd Semester AY 2013-2014**

Show all necessary solutions. Write legibly and box every final answer. Use black or blue pen only.

1) Use rectangular coordinates to evaluate the triple integral $\iiint_G y\,dV$, where G is the solid bounded by $2x + 3y + 2z = 6$ and the coordinate planes. (5 points)

2) Use cylindrical coordinates to find the volume of the solid in the first octant bounded by the planes $y = 2z$, $x = 0$, $z = 0$ and the cylinder $x^2 + y^2 = 4$. (5 points)

3) Use spherical coordinates to set up the iterated integral equal to the mass of the solid bounded above by the sphere $x^2 + y^2 + (z-2)^2 = 4$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$ if the density at any point (x, y, z) on the solid is $x + 1$. (5 points)

4) Let $\vec{F}(x, y, z) = 2ye^{2x}\,\vec{\boldsymbol{i}} + e^{2x}\,\vec{\boldsymbol{j}} + 3z^2\,\vec{\boldsymbol{k}}$. Show that $\vec{F}$ is conservative and use a potential function for $\vec{F}$ to find the value of the line integral $\int_C F \cdot dR$, where C is any sectionally smooth curve from the point $(\ln 2, 1, 1)$ to the point $(\ln 2, 2, 2)$. (5 points)

5) Evaluate the integral $\int_C (x^2 + xy)\,dx + (y^2 - xy)\,dy$, where C consists of the line segment $y = x$ from the point $(0, 0)$ to the point $(2, 2)$ and the vertical line from $(2, 2)$ to $(2, 0)$. (5 points)

6) Use Green's Theorem to evaluate the line integral $\oint_C y^2\,dx + x^2\,dy$, where C is the closed curve determined by the x-axis, the line $x = 1$, and the curve $y = x^2$ traversed in counter clockwise direction. (5 points)

7) Let the surface S be given by $\vec{r}(u, v) = u\cos v\,\vec{\boldsymbol{i}} + u\sin v\,\vec{\boldsymbol{j}} + (v + 3)\,\vec{\boldsymbol{k}}$

   (a) Find $\vec{r}_u \times \vec{r}_v$ (2 points)

   (b) Evaluate the surface integral $\iint_S \sqrt{x^2 + y^2}\,d\sigma$. (3 points)

8) Let $\vec{F}(x, y, z) = -x\,\vec{\boldsymbol{i}} + (y + 2)\,\vec{\boldsymbol{j}} + z\,\vec{\boldsymbol{k}}$ be the velocity field of a fluid and let S be the portion of the plane $3x + 2y + z = 6$ **in the first octant**. Find the flux of $\vec{F}$ across S. (5 points)

Total: 40 points

Any form of cheating in examinations or any act of dishonesty in relation to studies shall be subject to disciplinary action.