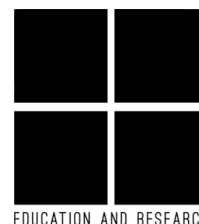


UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

erho.weebly.com | erhomyhero@gmail.com | f/erhoismyhero | @erhomyhero



Mathematics 55
Third Long Exam

M55_LE3_001
Elementary Analysis III
2nd Semester AY 2014-2015

1. The sequence $\{b_n\}_{n=1}^{+\infty}$ has a property that $b_1 + b_2 + \dots + b_n = \cos^{-1}\left(\frac{1}{n}\right)$ for all n . Determine if the series $\sum_{n=1}^{+\infty} b_n$ converges or diverges. (2 pts)

2. (a) Show that $\left\{\frac{1 \cdot 3 \cdots (2n-1)}{2 \cdot 4 \cdots (2n)}\right\}_{n=1}^{+\infty}$ (2 pts)

(b) Use the Monotone Convergence Theorem to show that the sequence in (a) converges. (2 pts)

3. (a) Verify that $\sum_{n=1}^{+\infty} \frac{1}{n^2} e^{1/n}$ satisfies the requirements of the Integral Test. (3 pts)

(b) Use the Integral Test to determine if the series in (a) converges or diverges. (3 pts)

4. Given the series $\sum_{n=1}^{+\infty} \frac{1 + \sin^2(n)}{2n}$.

(a) Explain why the Comparison Test fails when the given series is compared with the series $\sum_{n=1}^{+\infty} \frac{1}{n}$ (2 pts)

(b) Use the Comparison Test to determine if the given series converges or diverges. (2 pts)

5. Determine if the series is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

(a) $\sum_{n=1}^{+\infty} \left[\left(\frac{7}{5}\right)^n + \left(\frac{1}{n}\right)^{7/5} \right]$ (3 pts) (b) $\sum_{n=1}^{+\infty} \left(\frac{\tan^{-1}(n)}{2}\right)^n$ (3 pts) (c) $\sum_{n=1}^{+\infty} (-1)^n \frac{2n}{2n\sqrt{n} + 1}$ (6 pts)

6. (a) Show that the radius of convergence of the power series $\sum_{n=0}^{+\infty} \frac{(2x + e)^n}{e^n + 1}$ is $\frac{e}{2}$. (4 pts)

(b) Find the interval of convergence of the power series in (a) (5 pts)

7. (a) Find the Taylor series of $\cos(x)$ centered at π . (4 pts)

(b) Use the Maclaurin series of $\cos(x)$ to find the series representation for $\int_0^1 \cos(x^3) dx$. (4 pts)

END OF EXAM
Total: 45 points