UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research
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EDUCATION AND RESEARCH

Mathematics 55
Third Long Exam

M55_LE3_001
Elementary Analysis III
2nd Semester AY 2014-2015

1. The sequence $\left\{b_n\right\}_{n=1}^{+\infty}$ has a property that $b_1 + b_2 + \ldots + b_n = \cos^{-1}\left(\frac{1}{n}\right)$ for all n. Determine if the series $\sum_{n=1}^{+\infty} b_n$ converges or diverges. *(2 pts)*

2. (a) Show that $\left\{\frac{1 \cdot 3 \cdots (2n-1)}{2 \cdot 4 \cdots (2n)}\right\}_{n=1}^{+\infty}$ *(2 pts)*

   (b) Use the Monotone Convergence Theorem to show that the sequence in (a) converges. *(2 pts)*

3. (a) Verify that $\sum_{n=1}^{+\infty} \frac{1}{n^2} e^{1/n}$ satisfies the requirements of the Integral Test. *(3 pts)*

   (b) Use the Integral Test to determine if the series in (a) converges or diverges. *(3 pts)*

4. Given the series $\sum_{n=1}^{+\infty} \frac{1 + \sin^2(n)}{2n}$.

   (a) Explain why the Comparison Test fails when the given series is compared with the series $\sum_{n=1}^{+\infty} \frac{1}{n}$ *(2 pts)*

   (b) Use the Comparison Test to determine if the given series converges or diverges. *(2 pts)*

5. Determine if the series is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

   (a) $\sum_{n=1}^{+\infty} \left[\left(\frac{7}{5}\right)^n + \left(\frac{1}{n}\right)^{7/5}\right]$ *(3 pts)* (b) $\sum_{n=1}^{+\infty} \left(\frac{\tan^{-1}(n)}{2}\right)^n$ *(3 pts)* (c) $\sum_{n=1}^{+\infty} (-1)^n \frac{2n}{2n\sqrt{n}+1}$ *(6 pts)*

6. (a) Show that the radius of convergence of the power series $\sum_{n=0}^{+\infty} \frac{(2x+e)^n}{e^n + 1}$ is $\frac{e}{2}$. *(4 pts)*

   (b) Find the interval of convergence of the power series in (a) *(5 pts)*

7. (a) Find the Taylor series of $\cos(x)$ centered at π. *(4 pts)*

   (b) Use the Maclaurin series of $\cos(x)$ to find the series representation for $\int_0^1 \cos(x^3)\, dx$. *(4 pts)*

END OF EXAM
Total: 45 points