M17_LE4_001

Mathematics 17 — College Algebra and Trigonometry

Fourth Long Examination — First Semester, AY 2012-2013

I. Answer the following questions.   *1 point each*

1. What is the range of $y = 2 - \sin(4x - \pi)$?

2. Why is sin 46° = cos 44°?

3. In what quadrant does an angle θ lie if cot θ is positive but sin θ is negative?

4. What is the largest value of $g(x) = -3\sin(4x + 5)$?

5. Arrange the following from smallest to largest: sin 1, sin 2, sin 3.

6. What is the exact value of (1 – tan 1°) (1 – tan 2°) … (1 – tan 44°) (1 – tan 45°)?

II. Do as indicated.

1. Given: $\cos 2\alpha = -\dfrac{3}{5}$, $\sin\beta = \dfrac{3}{4}$, $\dfrac{\pi}{2} \le \alpha, \beta \le \dfrac{3\pi}{4}$. Find:

    a. $\cos\dfrac{\alpha}{2}$ *3 points*

    b. $\sin\alpha\sin\beta$ *1 point*

    c. $\sec(2\alpha - \beta)$ *2 points*

2. Given α, β are measures of acute angles in standard position with $A(2,1)$ on the terminal side of α and $B(3,1)$ on the terminal side of β. Find:

    a. $\cot(\alpha - \beta)$ *2 points*

    b. $\tan(\alpha + \beta)$ *2 points*

3. Find the exact value of $\sin\dfrac{7\pi}{8}$.   *2 points*

III. Given: $f(x) = 1 + 3\sin\left(\dfrac{1}{2}x - \dfrac{\pi}{4}\right)$.

1. Find the amplitude, the period, the phase shift, the vertical shift and the range of f.   *5 points*

2. Sketch the graph of f over one period.   *1 point*

IV. Prove the following identities.   *4 points each*

1. $\dfrac{1 + \cot^3 t}{1 + \cot t} = \csc^2 t - \cot t$

2. $\tan^2\theta + \sec\theta\tan\theta = \dfrac{\sin\theta}{1 - \sin\theta}$

V. Find the values of $x, x \in [0, 2\pi)$, in the following equations.   *4 points each*

1. $\sin^2 x = \cos^2 x$

2. $\tan 2x = \tan 4x$

END OF EXAM