UP SCHOOL OF STATISTICS STUDENT COUNCIL
EDUCATION & RESEARCH
PHILOS SOPHIA

M17_LE4_002

Mathematics 17 — College Algebra and Trigonometry
Fourth Long Examination — First Semester, AY 2007-2008

I. TRUE or FALSE *(1 pt each)*

1. The equation $9\cos^2\theta - 16 = 0$ has one real solution.
2. The graph of $y = \cos(\sin x)$ is symmetric with respect to the y-axis.
3. $\sin 5$ is negative.
4. The graph of $f(x) = \cot x$ has no y-intercepts.
5. If $\theta \in \mathbb{R}$ and $P(\theta) = (x, y)$, then $P(\theta, -\pi) = (-y, -x)$.

II. Multiple Choice. Write the CAPITAL letter of your answer. *(2 pts each)*

1. For any $\theta \in \mathbb{R}$, $\tan\left(\frac{x}{2} - \theta\right)$ is equal

   A. $-\tan\theta$ B. $\tan\theta$ C. $\cot\theta$ D. $-\cot\theta$

2. The distance between the point $Q(\cos 5, \sin 5)$ and the origin is

   A. 5 C. 1
   B. $5\sqrt{2}$ D. not possible to determine

3. A central angle subtends an arc length of 12 in a circle of diameter equal to 4. What is the measure of the central angle?

   A. 6° B. $\left(\frac{\pi}{30}\right)^\circ$ C. $\left(\frac{1080}{\pi}\right)^\circ$ D. $(12\pi)^\circ$

4. If the point with the coordinates (-2, 1) lies on the terminal side of the angle in standard position with measure α, then what is the value of $\sec\alpha$?

   A. $-\frac{\sqrt{5}}{2}$ B. $\frac{2\sqrt{5}}{5}$ C. $\frac{\sqrt{5}}{5}$ D. -5

5. Two coterminal angles in radian measure differ by

   A. multiples of π C. odd multiples of 2π
   B. odd multiples of π D. even multiples of π

III. Do as indicated.

A. Find the exact value of the following: *(3 pts each)*

1. $\sin 105^\circ \cos(2(262.5^\circ)) - \sin(2(262.5^\circ)) \cos 105^\circ$
2. $\cot 75^\circ + \tan 15^\circ$
3. $\dfrac{\sin^3\frac{\pi}{12} - \cos^3\frac{\pi}{12}}{\sin\frac{\pi}{12} - \cos\frac{\pi}{12}}$

B. Given that $\csc\alpha = -\frac{13}{5}$, P(α) in QIV; and $\sec\beta = -\frac{5}{3}$, $\pi \le \beta \le \frac{3\pi}{2}$. Find: *(3 pts each)*

1. $\sec\left(\frac{5\pi}{3} + \alpha\right)$
2. $\sin\frac{\alpha}{2}$
3. $\cot 2\beta$

C. Given the equation $y = -3\sin\left(\frac{x}{2} - 5\right) + 3$.

1. Sketch the graph on the interval $10 \le x \le 4\pi + 10$. Label important points. *(3 pts)*
2. Give the period, maximum, minimum, and amplitude of the function. *(2 pts)*
3. What are the zeros of the function? *(2 pts)*
4. Give the displacement of the graph. *(2 pts)*

D. Prove the following identities: *(4 pts each)*

1. $(1 + \sin x + \cos x)^2 = 2(1 + \sin x)(1 + \cos x)$
2. $\dfrac{\sec^4 x + \tan^4 x}{\sec^2 x \tan^2 x} - \dfrac{\cos^4 x}{\sin^2 x} = 2$

END OF EXAM