Mathematics 17 Fifth Long Examination

I. Write TRUE if the statement is correct, and write FALSE otherwise.

UP SCHOOL OF STATISTICS STUDENT COUNCIL

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1. The graph of $y = \tan^{-1} x$ does not intersect the line $y = \frac{\pi}{2}$.

Education

- 2. The equation $\sin x \cos x = 1$ has a solution in \mathbb{R} .
- 3. If $y \in [0,1]$, then $0 \le Arc \sin y \le \frac{\pi}{2}$.
- 4. There exists a triangle whose interior angles are all less than 60° .
- 5. If $\omega \in \mathbb{C} \setminus \{1\}$ and ω is a cube root of one, then $1 + \omega + \omega^2 = 0$.

II.

1. Evaluate:
$$\tan\left(Arc\cos\left(-\frac{12}{13}\right) + Arc\sin\left(\frac{3}{5}\right)\right)$$
. 4 points

- 2. Given: $z_1 = 3cis65^\circ$, $z_2 = \sqrt{2} + \sqrt{2}i$ and $z_3 = 2cis210^\circ$. Find:
 - (a) the imaginary part of $z_2 + \overline{z_3}$. 3 points

(b)
$$\frac{(z_1)^3 \cdot z_2}{z_3}$$
 (Express your answer in rectangular form.) 4 points

III.

1. Solve for all
$$x \in \mathbb{R}$$
 such that $Arc\cos(1-2x) + Arc\tan(-1) = Arc\sin\left(\sin\frac{11\pi}{12}\right)$. 4 points

2. Solve for all
$$x \in [0, 2\pi)$$
 such that $\cos 2x - 4\cos x = 2\sin^2 x$. 5 points

- 3. Solve for all $x \in \mathbb{C}$ such that $z^4 = 8\sqrt{2}(1-i)$. Leave your answer in polar form. 5 points
- IV. Solve the following problems completely.
 - 1. Along a street in a shopping district, Nobita observed that the angle of elevation of a tower is 30° . Walking 40 m towards the tower and stopping at a noodle shop, Nobita finds that the new angle of elevation of the tower is 60° . Find the height of the tower and the distance of the noodle shop from the base of the tower.
 - 2. Find the measures of the interior angles of a triangle with dimensions 6 by 10 by 14. (*To simplify your solution, first find the angle opposite the longest side.*)

END OF EXAM





5 points each

1 point each