UP SCHOOL OF STATISTICS STUDENT COUNCIL

EDUCATION & RESEARCH

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M53_FIN_001

Mathematics 53 — Elementary Analysis I

Final Examination — Second Semester, AY 2013-2014

I. True or False. Write T if the statement is always true. Otherwise, write F. (*1 point each*)

1. The function $f(x) = 4x^3 - 12x^2$ has a horizontal tangent line at the point (2, -16).
2. If $f''(a) = 0$, then f has an inflection point at $x = a$.
3. The graph of the function $g(x) = \cosh x + \sinh x$ is symmetric with respect to the y-axis.
4. If f is differentiable on $[a, b]$, then f is integrable on $[a, b]$.
5. The function $f(x) = \ln\left(\frac{5-x}{5+x}\right)$ is defined for all $x < 5$.

II. Consider

$$f(x) = \begin{cases} \cos x, & x < 0 \\ \dfrac{x \ln x}{1-x}, & 0 < x < 1 \\ -1, & x = 1 \end{cases}$$

Discuss the continuity of f on $[0,1]$. (*5 points*)

III. Solve for $\frac{dy}{dx}$. Do not simplify. (*4 points*)

1. $y = \sqrt[5]{\dfrac{4^x \coth(e^{-x})}{\sin(x^2 \ln x)}}$
2. $e^y = \cosh(e^x + \log_3 y) - 2^{x^2}$

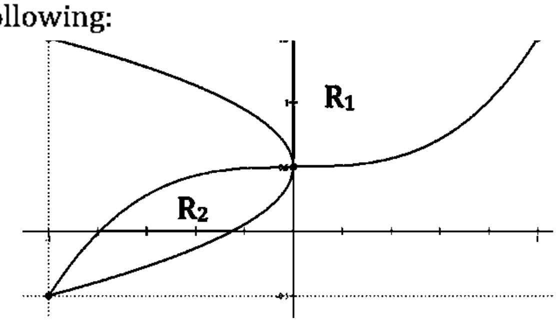
IV. Find the limits. (*4 points*)

1. $\lim\limits_{x \to 1^+} \left(\frac{1}{\ln x} - \frac{1}{\cosh^{-1} x}\right)$
2. $\lim\limits_{x \to 0^+} (e^x + \sinh x)^{\cot x}$

V. Evaluate the following integrals. (*5 points*)

1. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\ln^2(\sin x)}{\tan x} dx$
2. $\int \frac{2^x}{(2^x+1)\sqrt{4^x+2^{x+1}-8}} dx$
3. $\int \frac{dx}{x+4\sqrt{x}+13}$

VI. Consider the region enclosed by the curves $y = x^3 + \frac{1}{2}$ and $x = -\left(y - \frac{1}{2}\right)^2$. A sketch of the graph is shown on the right. Set-up the integrals representing the following:



1. The perimeter of the region. (*3 points*)
2. The total area of the regions R_1 and R_2. (*4 points*)
3. The volume of the solid generated when R_1 is rotated about the line $y = -\frac{1}{2}$ using the method of Washers. (*4 points*)
4. The volume of the solid generated when R_2 is rotated about the line $x = -1$ using the method of Cylindrical Shells. (*4 points*)

VII. Solve the following problems completely.

1. Find the equation of the line normal to the curve $y = \tanh(\sin^{-1}(x - \sqrt{2}))$ at the point where $x = \sqrt{2}$. (*4 points*)
2. A particle moves along the curve $y = \ln x$ so that its abscissa is increasing at a rate of 2 units per second. At what rate is the particle moving away from the origin as it passes through the point $(e, 1)$? (*5 points*)
3. Find the area of the largest rectangle that can be inscribed in the region bounded by the curve $y = e^{-x}$, the line $x = 0$ and the positive x-axis. (*5 points*)

END OF EXAM

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