M53_FIN_002

Mathematics 53   Elementary Analysis I

Final Examination   Second Semester, AY 2013-2014

I. Evaluate the following limits. (*5 points each*)

1. $\lim\limits_{x \to -\infty} \left(\sqrt{4x^2 - 3x + 1} + 2x\right)$

3. $\lim\limits_{x \to 0^+} (\sin x)^{\tan x}$

2. $\lim\limits_{x \to 0} 10^{\left[\frac{\tan^{-1}(3x)}{\ln(2x+1)}\right]}$

II. Differentiate the following to solve for $\frac{dy}{dx}$. No need to simplify. (*5 points each*)

1. $y = \dfrac{5^{x^2}(5e^x + \csc x^2)}{\log_3(\sin^{-1}\sqrt{x})}$

2. $y = x^{\cot^{-1}\left(1 - \frac{x}{y}\right)}$

III. Given that $f(x) = \left(\dfrac{x-1}{x-2}\right)^2$, $f'(x) = \dfrac{-2(x-1)}{(x-2)^3}$, and $f''(x) = \dfrac{2(2x-1)}{(x-2)^4}$.

1. Using limits, determine the equations of the horizontal and vertical asymptotes of the graph of f. (*2 points*)
2. Find the intervals where f is increasing or decreasing. (*2 points*)
3. Find the intervals where the graph of g is concave up or concave down. (*2 points*)
4. Find the coordinates of the critical and inflection points of the graph of f. (*2 points*)
5. Sketch the graph of f, with emphasis on concavity. (*2 points*)
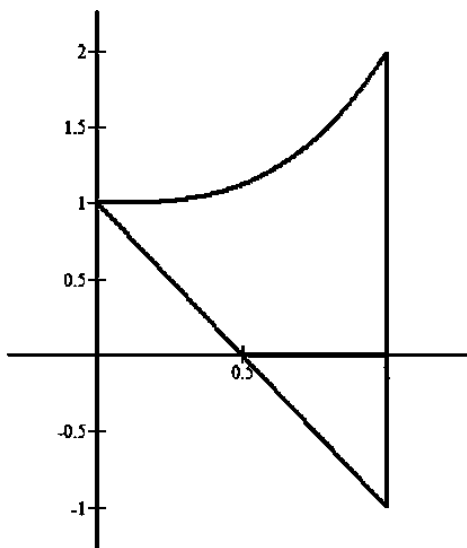
IV. Evaluate the following integrals. (*6 points each*)

1. $\int \dfrac{x}{\sqrt{x^2 + 6x}}\,dx$

3. $\int \dfrac{1-x}{4x^2 + 9}\,dx$

2. $\int \dfrac{1}{\sqrt{e^{2x} - 25}}\,dx$

4. $\int_0^{\frac{\pi}{2}} \sin^3 x \cdot \sqrt[5]{\cos x}\,dx$

V. Let R be the region bounded by the curves $y = x^3 + 1$, $y = 1 - 2x$, and $x = 1$. Setup the definite integral that will yield the following:

1. The area of area of R using vertical strips. (*2 points*)
2. The perimeter of R, in terms of *y*. (*3 points*)
3. The volume of the solid obtained by revolving R about the line *x* = 2, by the method of Cylindrical Shells. (*3 points*)
4. The volume of the solid obtained by revolving R about the line *x* = -3, by the method of Washers. (*3 points*)

VI. Answer completely the following word problems. (*5 points each*)
1. Find the height of a right circular cone, whose slating side is $\sqrt{3}$ feet long, having the largest possible volume.
2. A light is hung 15 feet atop a lamp post on a level ground. If a man 6 feet tall is walking away from the light at a rate of 6 feet per second, how fast is his shadow on the ground lengthening?

END OF EXAM