



Mathematics 53
Final Examination

M53_FIN_003
Elementary Analysis I
Second Semester, AY 2013-2014

I. Find $\frac{dy}{dx}$. No need to simplify. (5 points each)

1. $e^y \log(2 + \sinh^{-1} x) + x^3 3^x + e^2 = \tan^{-1} \left(\frac{y}{x} \right)$

2. $y = \frac{(\cos x)^{\sin x} \cdot \sqrt[3]{(5x^2+1)^2}}{\ln(2^x)}$

II. Evaluate the following limits. (4 points each)

1. $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2+1}-x}{x}$

2. $\lim_{x \rightarrow +\infty} \frac{\operatorname{sech} x}{1 - \tanh x}$

3. $\lim_{x \rightarrow 0^+} (\sin^{-1} x)^x$

III. Given:

$$f(x) = \begin{cases} \frac{e^{2x} - 1}{2 \sin x}, & x < 0 \\ \frac{|1-x|}{1-x}, & x \geq 0 \end{cases}$$

Discuss the continuity of f at $x = 0$ and $x = 1$. Identify each type of discontinuity, if any.

IV. Find the following antiderivatives. (5 points each)

1. $\int 5^x \ln(x^2) (\ln(x^2) + 2) dx$

2. $\int \frac{\sin \sqrt[3]{x} (1 + \sec \sqrt[3]{x})}{(\sqrt[3]{x})^2} dx$

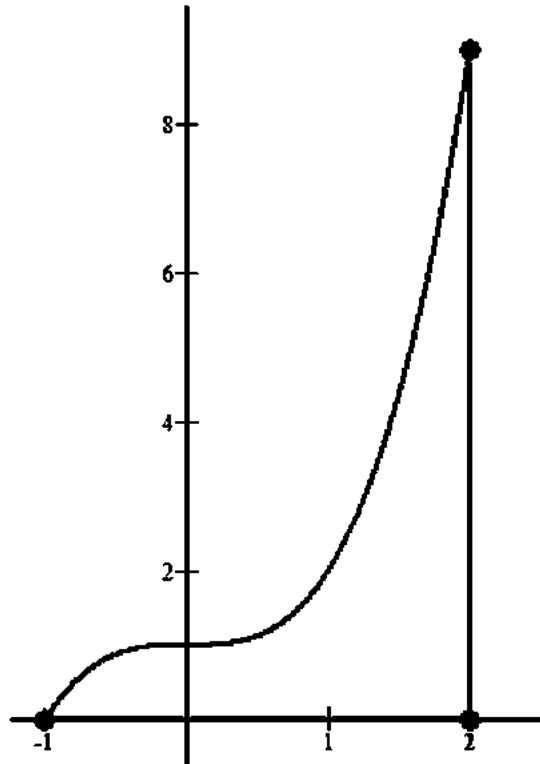
3. $\int \frac{x-5}{\sqrt{25-9x^2}} dx$

V. Evaluate the definite integral: $\int_0^1 \frac{1-5e^{5x}}{(e^{5x}-x)^2} dx$. (5 points)

VI. Word Problems. (5 points each)

- Determine the interval/s on which the graph of $y = 20x + 5x^4 - x^5$ is concave downward.
- A car driver travelling at a constant velocity of 90 feet per second along a straight road applies the brake such that the car has a constant negative acceleration of 15 feet per second squared.
 - Find an expression for $s(t)$ where s is in feet measured from the point where the brake was first applied ($t = 0$).
 - How long will it take for the car to come to a stop?
 - How far will the car travel before stopping?
- A piece of wire 80 centimeters long is bent to form a rectangle. Find the dimensions of the rectangle so that its area is as large as possible.
- Suppose that John's tumor is spherical in shape and its radius is decreasing at the rate of $\frac{1}{160}$ centimeters per day. What is the rate of decrease of the volume of his tumor when its radius is 0.4 centimeters?

VII. Let R be the region bounded by the graphs of $y = x^3 + 1$, $x = 2$, and the x -axis. Set-up the definite integral equal to



1. The area of R using horizontal strips. (3 points)
2. The volume of the solid generated when R is revolved about the line with equation $x = 2$ using the method of Disks. (3 points)
3. The volume of the solid generated when R is revolved about the line with equation $x = 2$ using the method of Cylindrical Shells. (3 points)
4. The perimeter of the boundary of R , in terms of x . (3 points)

END OF EXAM