M53

Mathematics 53
First Long Examination

Elementary Analysis I
First Semester, AY 2012-2013
I. Evaluate the following limits.

1. $\lim _{x \rightarrow 2 / 3} \frac{27 x^{3}-8}{3 x^{2}-5 x+2}$
(4 points)
2. $\lim _{x \rightarrow-\infty} \frac{x-\sqrt{4 x^{2}+1}}{6 x-7}$
3. $\lim _{\theta \rightarrow 0} \frac{\tan ^{2} 3 \theta}{\theta^{3}+2 \theta^{2}}$
(5 points)
4. $\lim _{x \rightarrow 1 / 2^{-}}\left(\frac{1}{|2 x-1|}+\frac{1}{4 x^{2}-1}\right)$
(4 points)
(5 points)
II. Given: $f(x)= \begin{cases}\frac{x-\llbracket x+1 \rrbracket}{x^{2}-3 x+2} & , x<1 \\ \frac{6-3 \sqrt{x}}{x-4} & , x \geq 1\end{cases}$

Discuss the continuity of $f$ at $x=0,1,4$. If discontinuous, classify the discontinuity as either removable, jump essential or infinite essential.
(12 points)
III. Do as indicated.

1. Use the Intermediate Value Theorem to show that $f(x)=2 x^{3}-3 \sin x-9$ has at least one real zero between 0 and $\pi$.
2. Use the Squeeze Theorem to evaluate $\lim _{x \rightarrow+\infty} \frac{x \cos 5 x}{3 x^{2}-1}$.
3. Suppose that the graph of $y=f(x)$ is given by the following figure:

(a) Find $\lim _{x \rightarrow-\infty} f(x)$.
(b) Find $\lim _{x \rightarrow-2^{+}} f(x)$.
(c) Find $\lim _{x \rightarrow 4} f(x)$.
(d) Find all values $a$ such that $f$ is discontinuous at $x=a$.
