M53_LE1_002

Mathematics 53 — Elementary Analysis I

First Long Examination — First Semester, AY 2012-2013

I. TRUE or FALSE. Write True if the statement is correct. Otherwise, write False. *(1 point each)*

1. If the functions *f* and *g* are both discontinuous at *x* = a, then *f* + *g* is discontinuous at the same point.
2. Given *h(x)* = $[\![x]\!]$, the limit of the function *h* from the left is not equal to the limit from the right for any a $\in \mathbb{R}$ because it contains infinitely jump discontinuities.
3. The limit of the function *p(x)* = $\dfrac{1}{x^2}$ exists.
4. Polynomial functions and Rational functions are continuous everywhere.
5. The Intermediate Value Theorem states that if *f* is continuous on a closed interval [a,b] with *f(a)*$\neq$ *f(b)*. For every *k* between *f(a)* and *f(b)*, there exists *c* in [a,b] such that *f(c)* = *k*.

II. EVALUATE. Evaluate the following limits. *(2 points each)*

1. $\lim\limits_{x\to 1}\left[\dfrac{4x^4-5x^3+4}{(7x-6)^{10}}\cdot\dfrac{\left(\dfrac{1}{x^3}\right)^7}{9x^5-7x^4-6x+7}\right]$
2. $\lim\limits_{x\to 2}\dfrac{x^2-4}{\sqrt[3]{x-3}+1}$
3. $\lim\limits_{x\to -1^-}\left(\dfrac{[\![x+1]\!]}{x+1}+\dfrac{x^3+2x^2-5x+1}{x^3+2}\right)$
4. $\lim\limits_{y\to -\infty}\left(\sqrt{9y^2+4}-\sqrt{9y^2-2y}\right)$
5. $\lim\limits_{t\to 0^+}\dfrac{1}{t}\left(1-\dfrac{1}{\sqrt{2t+1}}\right)$
6. $\lim\limits_{m\to +\infty}\sqrt{m^2+3m}-m$
7. $\lim\limits_{x\to 2^-}\dfrac{x+[\![-x]\!]}{x^2-1-[\![3x-2]\!]}$
8. $\lim\limits_{x\to 2^-}\dfrac{x^2-\operatorname{sgn}(x^2-2x)}{[\![x-2]\!]\cdot|x+2|}$
9. $\lim\limits_{x\to 1}\dfrac{\sin(\sin(x))}{x}$
10. $\lim\limits_{x\to -\infty}\dfrac{2x^3+1+5\cos x}{3x^3}$

III. Do as indicated. *(2 points each)*

1. Sketch the graph of the function satisfying the following conditions: dom *f*: [-4,4], *f*(-4)= *f*(-2)=3, *f*(0)=1, *f*(2)=-1, *f*(4)=0, $\lim\limits_{x\to -4^+} f(x)=0$, $\lim\limits_{x\to -4} f(x)=1$, $\lim\limits_{x\to 0^-} f(x)=1$, $\lim\limits_{x\to 0^+} f(x)=4$, $\lim\limits_{x\to 2} f(x)=-1$ and $\lim\limits_{x\to 4^-} f(x)=0$.
2. Show that *g(x)*= x^3-3x+1 has a zero between 0 and 2.
3. State the Squeeze Theorem.
4. Given $f(x)=\begin{cases} x^2 & \text{if } x\le -2 \\ ax+b & \text{if } -2<x<2. \\ 2x-6 & \text{if } x\ge 2\end{cases}$ Find the value of a and b such that $\lim\limits_{x\to -2} f(x)$ and $\lim\limits_{x\to 2} f(x)$ exists.
5. Find the value of $\lim\limits_{x\to 0^+}\sin\left(\dfrac{1}{x}\right)$.

IV. Discuss the continuity of the function. Sketch the graph. *(5 points)*

$$F(x)=\begin{cases} |x+3| & ,x\le 0 \\ [\![2x]\!] & ,0<x<1 \\ \dfrac{x^2-5x}{x-5} & ,x\ge 1\end{cases}$$

END OF EXAM