



I. Write TRUE if the statement is correct. Otherwise, write FALSE. (1 point each)

1. If  $f$  is differentiable at  $x = a$ , then the tangent line to the graph of  $f$  at  $P(a, f(a))$  intersects the graph of  $f$  at a unique point.
2. If  $a \in \mathbb{R}$  and  $f(a) = 2$ , then  $f'(a) = 0$ .
3. For any  $x \in \text{dom sec}$ ,  $\lim_{\Delta x \rightarrow 0} \frac{\sec(x + \Delta x) - \sec x}{\Delta x} = \sec x \tan x$ .
4. It is impossible for a function to be both differentiable and discontinuous at  $x = a$ .
5. If  $f''(x) = g''(x)$  for every  $x \in \mathbb{R}$ , then  $f'(x) = g'(x)$  for every  $x \in \mathbb{R}$ .

II. Find  $\frac{dy}{dx}$ . There is no need to simplify. (5 points each)

1.  $y = \frac{\sqrt[3]{3x^4 - 2} + \frac{4}{x^2}}{\csc(x) - \sqrt{5}}$
2.  $\cot(x^2y) = 6\sqrt{x} + y^3$

III. Solve the following problems completely.

1. Given:  $f(2) = 4$  and  $f'(x) = \sin^2\left(\frac{\pi}{3}x\right)$ 
  - (a) Find an equation of the normal line to the graph of  $f$  at  $x = 2$ . (3 points)
  - (b) Use local linear approximation to estimate the value of  $f(1.98)$ . (Give your answer in decimal form.) (2 points)
  - (c) Determine  $f'''(x)$ . (There is no need to simplify.) (4 points)
2. Determine whether the following function is differentiable at  $x = 1$ : (5 points)

$$f(x) = \begin{cases} \tan(1-x), & x > 1 \\ x^2 - x, & x \leq 1 \end{cases}$$

3. The Invisible Pink Unicorn (IPU) is flying along a horizontal line such that its position with respect to a point  $A$  on the line at any time  $t$  is  $s(t) = (t^4 - 2t^3 + t^2 - 3)$  meters.
  - (a) Determine all intervals when the IPU is moving toward the left. (4 points)
  - (b) The **jerk** of an object moving along a line is defined as the instantaneous rate of change of its acceleration. Find the position of the IPU with respect to  $A$  when its jerk was equal to 12 meters/second<sup>3</sup>. (3 points)
4. Mona, who is 6 feet tall, is jogging between a wall and a light source that is on the ground and 30 feet from the wall. When her shadow on the wall was 9 feet high, it was found that her shadow was getting taller at a rate of 3 feet per second. How fast and in what direction was Mona jogging at that instant? (4 points)