M53_LE2_002

Mathematics 53 — Elementary Analysis I

Second Long Examination — First Semester, AY 2012-2013

I. Write TRUE if the statement is always true. Otherwise, write FALSE. *(1 pt. each)*

1. By definition, the derivative of a function $f(x)$ is $f'(x)=\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$.
2. All functions that are not differentiable are discontinuous but not all discontinuous functions are not differentiable at some $x = x_0$.
3. If f and g are differentiable functions where $f(x)\neq g(x)$, then $f'(x)\neq g'(x)$.
4. $f^{(23)}(x)=-\cos(x)$ given $f(x)=\sin(x)$.
5. $\frac{d}{dy}[\sec^2(y^2+3)]=4y\sec^2(y^2+3)\tan(y^2+3)$.

II. Find $\dfrac{dy}{dx}$. Do not simplify. *(5 pts. each)*

1. $y=\cot\left(\dfrac{\sqrt[3]{\sec^2(3x-1)}}{\csc(x^2)-\sqrt{\sin\pi}}\right)-\tan(\frac{\pi}{4})^{\cos\pi}$
2. $\sin^2(x^3+y)+\cot\left(\frac{x}{y}\right)=3x^3$

III. Do as indicated. *(3 pts. each)*

1. Find the equation of the normal line to the graph of $y^3-xy^2+\cos(xy)=\csc^2(\frac{\pi}{4})$ at (0,1).
2. Let a and b be real numbers and consider $f(x)=\begin{cases}-\dfrac{1}{x^2+1} & ,x\leq 0\\ bx^3-ax^2+2 & ,0<x<2\\ a\sec(3x-6) & ,x\geq 2\end{cases}$

   Find the values of a and b that would make f differentiable at x=2.
3. Estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemisphere dome with diameter 50 m.
4. Find the slope of the tangent line at each point of the graph of $y=x^4+x^3-3x^2$ where the rate of change of the slope is equal to zero.
5. If the directed distance of a particle moving along the x-axis from the origin is $s(t)=(t-1)^3(t-3)$, determine when it changes direction.

IV. Solve the following problems completely. Show your solutions. *(5 pts. each)*

1. A bird and a worm are on the ground and are $20\sqrt{3}$ feet apart. Deciding it was not hungry, the bird flies away from the worm on a straight path that makes an angle of 60^o with the ground. If the bird is flying at a constant rate of 18 feet per second, at what rate is the angle of elevation of the bird from the worm changing at the instant when the bird is 30 feet above the ground? Indicate whether the angle is increasing or decreasing at that instant.
2. A particle is moving along a horizontal line, and its equation of motion is given by $s(t)=t^3-6t^2-9t+54$ where s is the directed distance (in feet) of the particle from the origin and t is in seconds. At what interval is the particle slowing down?

END OF EXAM