M53_LE3_001

Mathematics 53 — Elementary Analysis I
Third Long Examination — First Semester, AY 2012-2013

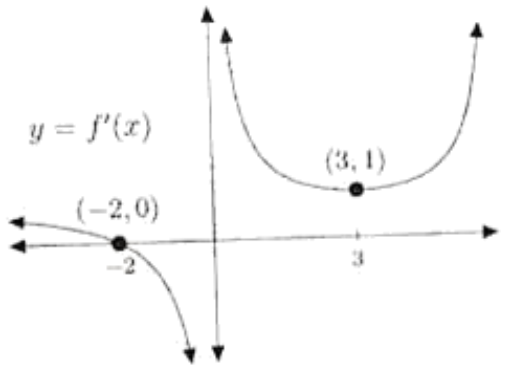
I. Write TRUE if the statement is correct. Otherwise, write FALSE. *(1 point each)*

1. If f has a relative maximum at $x = c$ on (a, b), then f has an absolute maximum at $x = c$.
2. Rolle's Theorem is applicable to the function $f(x) = \tan x$ on $[0, \pi]$.
3. A rational function may have two distinct oblique asymptotes.

II. Given $f(x) = \dfrac{x^2+4}{(x-2)^2}$, $f'(x) = \dfrac{-4x-8}{(x-2)^3}$ and $f''(x) = \dfrac{8x+32}{(x-2)^4}$.

1. Find the domain and the intercepts of f. *(2 points)*
2. Using limits, find the equations of all linear asymptotes of the graph of f. *(3 points)*
3. Find the critical numbers and possible points of inflection of f. *(2 points)*
4. Construct a table of signs for f' and f''. Indicate the intervals on which f is increasing or decreasing, concave up or concave down, and specify the points where f has relative extrema and points of inflection, if any. *(7 points)*
5. Sketch the graph of f and label all important points such as intercepts, relative extrema and points of inflection. *(3 points)*

III. Refer to the graph of f' below, and assume that f is a function continuous everywhere. Select the correct answer based from the given choices. *(1 point each)*



1. f is [*increasing / decreasing*] on $(-\infty, -2)$.
2. f has [*a relative maximum / a relative minimum / a point of inflection*] at $x = 0$.
3. f is [*concave up / concave down*] on $(0, 3)$.
4. f has [*a relative maximum / a relative minimum / a point of inflection*] at $x = 3$.

IV. Do as indicated.

1. (a) State the Mean Value Theorem. *(1 point)*
   (b) Suppose g is a differentiable function such that $3 \leq g'(x) \leq 5$ for every $x \in \mathbb{R}$. Show that $24 \leq g(5) - g(-3) \leq 40$ using the Mean Value Theorem. *(2 points)*
2. The distance between two particles moving along a horizontal coordinate line is given by $d(t) = 2 + \cos t - \sin^2 t$ where d is the distance measured in feet at the end of t seconds. Over the time interval $[0, \pi]$, when will the two particles be closest to one another? When will they be farthest from each other? *(4 points)*
3. A cylinder is to be formed by revolving a rectangle about one of its edges. If the rectangle's diagonal measures 3 inches, what must be the dimensions of the rectangle that will result in a cylinder of maximum volume? *(4 points)*

END OF EXAM