M53_LE3_002
Mathematics 53
Third Long Examination
I. Write TRUE if the statement is always true. Otherwise, write FALSE. (1 pt. each)

1. Let $g$ be a function whose $g^{\prime}(x) \geq-7 \forall x \in[0,3]$ and such that it crosses the point of origin. The least possible value for $g(3)$ is -21 .
2. If $f^{\prime}(c)=0$ or $f^{\prime}(c)$ does not exist, then the graph of the function $f$ has a point of inflection at $P(c, f(c))$.
3. The absolute maximum or minimum of a function on any interval is unique.
4. The graph of a function $f$ cannot touch its respective asymptote.
5. If $f$ is a function such that $f^{\prime}(c)=0=f^{\prime \prime}(c)$ on some open interval containing $x=c$, then the function may or may not have a relative extrema at $x=c$.
II. Do as indicated.
6. Verify that the function $f(x)=x(x+6)^{1 / 2}$ satisfy the conditions of Rolle's Theorem on the interval $[-6,0]$ and find all numbers that satisfy its conclusion. (2 points)
7. Show that the function $g(x)=x(x+2)^{-1}$ satisfy the hypotheses of Mean Value Theorem on the interval $[1,4]$ and find all numbers that satisfy its conclusion. (2 points)
8. Given the function: $F(x)=\left\{\begin{array}{ll}4-(x+5)^{2} & x<-4 \\ 12-(x+1)^{2} & -4 \leq x\end{array}\right.$. Find the relative extrema of the function and determine when the function is increasing or decreasing. (4 points)
9. Find all points of inflection of $h(x)=2 \sin 4 x, x \in[0, \square \square \square \square \square$ and look for the intervals where the function is concave up or concave down.
(3 points)
10. Sketch the graph of $G(x)=\frac{x^{3}}{(x+1)^{2}}$.
(5 points)
11. Given that $f$ if continuous everywhere and that the graph of $f^{\prime}$ is as shown. Sketch a possible graph of $f$.
(5 points)
12. Find absolute extremum values of the function $j(x)=16 x^{-1}-$ $x^{2} \forall x \leq-1$.
(4 points)

III. Problem Solving.
13. Find the smallest and largest distance of $P(3,-4)$ from a point on the circle with radius 3 units and center at the origin.
(5 points)
14. A rectangular open tank of volume 125 cubic meters is to have a square base. The cost per square meter for the bottom is Php 200.00 and Php 100.00 for the others. What would be the dimensions of the tank so that the cost of the materials is of least?
(5 points)
