M53_LE4_002

Mathematics 53   Elementary Analysis I

Fourth Long Examination   First Semester, AY 2012-2013

I.   Write TRUE if the statement is correct. Otherwise, write FALSE.   [*1 point each*]
1. All differentiable functions are integrable functions.
2. As the number of rectangles goes to infinity, the length of each subinterval goes to 0.
3. The definite integral of $f(x) = 2x^{-4}$ on the interval [-2, 2] is $-\frac{1}{6}$.

II.   Evaluate.   [*2 points each*]

1. $$\int \frac{x^3}{\sqrt{1-2x^2}}\, dx$$

2. $$\int (\tan 2\theta + \cot 2\theta)^2\, d\theta$$

3. $$\int^3 \sqrt[3]{4y^2 + 12y - 9}\, dy$$

4. $\frac{d}{dx}\int_{2-x^2}^{2+x^2} (t^2-4)dt$

5. $D_x\left(\int_{\tan x}^{0} \sin w \cos w\, dw\right)$

6. $\int_4^{16}\left[D_x \int_5^x (2\sqrt{t} - 1)dt\right]dx.$

III.   Do as indicated.
1. Find the average value of $f(x) = |3x - 6| - 1$ in the interval [0,3]. Find the value of x at which the average value occurs.   [*3 points*]
2. Let R be the region bounded by the curves: $2y = x + 3$, $y^2 + x = 0$ and $y^2 - 4x = 0$.
   a. Find the area of the region.   [*3 points*]
   b. Find the perimeter of the region.   [*3 points*]
   c. Set up the integral to find the volume of the solid generated when the region is revolved about the line $2y = x + 3$.   [*4 points*]
   d. Set up the integral to find the volume of the solid generated when the region is revolved about the line $y = 3$ using:
      (i)   disk-washer method   [*4 points*]
      (ii)   cylindrical shell method   [*4 points*]

IV.   Solve the following problems completely.
1. As Kenneth is climbing a mountain, he looks up and sees that a boulder becomes dislodged from the cliff. If his head is 200 ft below the base of the boulder at that instant, how much time Kenneth have to get out of the way? If he was not able to get out of the way in time (hopefully), with what speed will the boulder hit Kenneth and let him die?   [*4 points*]

END OF EXAM