M53_LE5_001

Mathematics 53 — Elementary Analysis I

Fifth Long Examination — First Semester, AY 2012-2013

I. Find dy/dx. There is no need to simplify.

1. $y=\dfrac{[\cot^{-1}(5^x)]^2}{1-\cosh(\sqrt{x})}$ (*4 points*)

2. $y=\dfrac{\sqrt[4]{x^2+1}\cot^3(2x)}{\sinh^{-1}(x)}$ (Use logarithmic differentiation.) (*5 points*)

3. $y=\left[\sec\frac{1}{x}\right]^{\sec^{-1}(e^{4x})}$ (*5 points*)

II. Evaluate the following limits. (*4 points each*)

1. $\lim\limits_{x\to1}\dfrac{(\log x)^2}{x^3-3x+2}$

2. $\lim\limits_{x\to\infty}(1+\pi e^{-x})^{e^x}$

III. Perform the integration.

1. $\displaystyle\int\frac{2^{\sin x}\sin x+3^{-\sin x}\sec x}{3^{-\sin x}\tan x}dx$ (*5 points*)

2. $\displaystyle\int_0^{\ln 2}\tanh^2 x\,dx$ (*4 points*)

3. $\displaystyle\int\frac{x^3+x}{16x^4-1}dx$ (*5 points*)

IV. At any point (*x*,*y*) on the curve, the tangent line has slope equal to $\dfrac{3}{\sqrt{4x-x^2}}$. If the point (1,0) lies on the curve, determine its equation. (*4 points*)

END OF EXAM

Formulas:

$1-\tanh^2 x=\operatorname{sech}^2 x$

$1-\coth^2 x=-\operatorname{csch}^2 x$

$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$

$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$

$\sinh 2x=2\sinh x\cosh x$

$\cosh 2x=\cosh^2 x+\sinh^2 x=1+2\sinh^2 x=2\cosh^2 x-1$

$\sinh^{-1}x=\ln(x+\sqrt{x^2+1})$

$\cosh^{-1}x=\ln(x+\sqrt{x^2-1})$

$\tanh^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$\coth^{-1}x=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$\operatorname{sech}^{-1}x=\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$\operatorname{csch}^{-1}x=\ln\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{|x|}\right)$

$D_x(\sinh^{-1}x)=\frac{1}{\sqrt{x^2+1}}$

$D_x(\cosh^{-1}x)=\frac{1}{\sqrt{x^2-1}}$

$D_x(\tanh^{-1}x)=\frac{1}{1-x^2},|x|<1$

$D_x(\coth^{-1}x)=\frac{1}{1-x^2},|x|>1$

$D_x(\operatorname{sech}^{-1}x)=-\frac{1}{x\sqrt{1-x^2}}$

$D_x(\operatorname{csch}^{-1}x)=-\frac{1}{|x|\sqrt{x^2-1}}$

$\int\frac{1}{\sqrt{u^2+a^2}}du=\sinh^{-1}\left(\frac{u}{a}\right)+C=\ln(u+\sqrt{u^2+a^2})+C$

$\int\frac{1}{\sqrt{u^2-a^2}}du=\cosh^{-1}\left(\frac{u}{a}\right)+C=\ln(u+\sqrt{u^2-a^2})+C, u>a$

$\int\frac{1}{a^2-u^2}du=\begin{cases}\frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right)+C, |u|<a\\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right)+C, |u|>a\end{cases}=\frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right|+C$

END OF EXAM