UP SCHOOL OF STATISTICS STUDENT COUNCIL

EDUCATION & RESEARCH

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M53_LE5_002

Mathematics 53 — Elementary Analysis I

Fifth Long Examination — First Semester, AY 2012-2013

Evaluate the following derivatives, integrals and limits. *(4 points each)*

1. $D_x\left((\csc\sqrt[4]{x})^{(6^x+x^6)}+\log_3(\log_7(1-\cosh x))\right)$

2. $\dfrac{d}{dx}\left(\sqrt[5]{\dfrac{4^x\coth(e^{-x})}{\sin(x^2\ln(\frac{x}{2}))}}\right)$

3. $(\cot x^3)^e+\sqrt{5}^{\log_6 y}=7^{x^2y}+\pi^{e^2}$

4. $\displaystyle\int\frac{x^4-7x^2+2x-1}{x+3}dx$

5. $\displaystyle\int\frac{2x+6}{x^2+2x+8}dx$

6. $\displaystyle\int\frac{1}{x^{1/3}+x}dx$

7. $\displaystyle\int\frac{e^{2x}+e^x+1}{\sqrt{e^{2x}-4}}dx$

8. $\displaystyle\lim_{x\to\infty}\left(\frac{-3+2x}{5+2x}\right)^{1+2x}$

9. $\displaystyle\lim_{x\to1^+}\left(\frac{1}{\ln(x)}-\frac{1}{\cosh^{-1}(x)}\right)$

10. $\displaystyle\lim_{x\to0^+}(e^x+\sinh(x))^{\cot(x)}$

Formulas:

$$1-\tanh^2 x=\operatorname{sech}^2 x$$
$$1-\coth^2 x=-\operatorname{csch}^2 x$$
$$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$$
$$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$$
$$\sinh 2x=2\sinh x\cosh x$$
$$\cosh 2x=\cosh^2 x+\sinh^2 x=1+2\sinh^2 x=2\cosh^2 x-1$$
$$\sinh^{-1}x=\ln(x+\sqrt{x^2+1})$$
$$\cosh^{-1}x=\ln(x+\sqrt{x^2-1})$$
$$\tanh^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$
$$\coth^{-1}x=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$
$$\operatorname{sech}^{-1}x=\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$$
$$\operatorname{csch}^{-1}x=\ln\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{|x|}\right)$$

$$D_x(\sinh^{-1}x)=\frac{1}{\sqrt{x^2+1}}$$
$$D_x(\cosh^{-1}x)=\frac{1}{\sqrt{x^2-1}}$$
$$D_x(\tanh^{-1}x)=\frac{1}{1-x^2},|x|<1$$
$$D_x(\coth^{-1}x)=\frac{1}{1-x^2},|x|>1$$
$$D_x(\operatorname{sech}^{-1}x)=-\frac{1}{x\sqrt{1-x^2}}$$
$$D_x(\operatorname{csch}^{-1}x)=-\frac{1}{|x|\sqrt{x^2-1}}$$
$$\int\frac{1}{\sqrt{u^2+a^2}}du=\sinh^{-1}\frac{u}{a}+C=\ln(u+\sqrt{u^2+a^2})+C$$
$$\int\frac{1}{\sqrt{u^2-a^2}}du=\cosh^{-1}\frac{u}{a}+C=\ln(u+\sqrt{u^2-a^2})+C,u>a$$
$$\int\frac{1}{a^2-u^2}du=\begin{cases}\frac{1}{a}\tanh^{-1}\frac{u}{a}+C,|u|<a\\ \frac{1}{a}\coth^{-1}\frac{u}{a}+C,|u|>a\end{cases}=\frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right|+C$$

END OF EXAM