UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research

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EDUCATION AND RESEARCH

M54-FE-001

Mathematics 54 | Elementary Analysis II

Final Examination | First Semester, AY 2013 -2014

I. Write TRUE if the statement is always true. Otherwise, write FALSE. *1 point each*

1. The vectors <1,-1> and <-1,1> are parallel.
2. The polar curve $r = \sin 4\theta$ is a 4-petalled rose.
3. The equation $z^2 = x^2 + y^2 + 1$ represents a hyperboloid of two sheets.
4. The eccentricity of any hyperbola is greater than the eccentricity of any ellipse.
5. The curve with parametric equations $x = t^2 + 1, y = t^2$ is a parabola.
6. Let $r = f(\theta)$ be a polar curve where *f* is differentiable. The slope of the tangent line to the point where $\theta = 0$ is $f'(0)$.

II. Perform the following integration . *5 points each*

1. $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$
2. $\displaystyle\int_0^{\infty} xe^{-x}\,dx$

III. Find the area under the graph of $y = \sin^2 x$ from $x = 0$ to $x = \pi$. *4 points*

IV. Set-up a definite integral equal to the area outside the limaçon $r = 4 - 3\sin\theta$ but inside the circle $r = 5\sin\theta$. *4 points*

V. Let ℓ be the line with parametric equations $x = 3 + 2t, y = -1 + 2t, z = 2 - t$ and let Π be the plane $2x - y + 2z = 5$.

1. Show that the line is parallel to the plane. *3 points*
2. Find the distance of the line from the plane. *3 points*

VI. Let $\vec{R}(t)$ be a vector-valued function such that $\vec{R}(0) = \langle 1,-1,3\rangle, \vec{R}'(0) = \langle 1,2,-2\rangle, \vec{R}''(0) = \langle 2,0,1\rangle$

1. Find the tangent line to the graph of $\vec{R}(t)$ at $t = 0$. *3 points*
2. Find the curvature at $t = 0$. *4 points*

VII. A bee has velocity function $\vec{V}(t) = \langle -3\sin t, 4, 3\cos t\rangle$.

1. Find the acceleration of the bee at time $t = \pi$. *1 point*
2. Find the position function given that the bee is located at the point (0,1,3) at time $t = 0$. *3 points*
3. Find the distance travelled by the bee from $t = 0$ to $t = 2$. *3 points*

VIII. Let $f(x, y) = 9x^2 - y^2$.

1. Identify the surface $z = f(x, y)$. *1 point*
2. Sketch the level curve of $f(x, y)$ of height 36. *4 points*
3. Find an equation of the tangent plane to $z = f(x, y)$ at the point (1,-1,8). *3 points*
4. If, in addition, $x = uve^u$ and $y = u^2v + u\ln v$, use the chain rule to find $\frac{\partial f}{\partial u}$. *4 points*

IX. A rectangular field measures 300 m by 400 m. If a path of uniform 1 m width is constructed around it, use differentials to estimate the area covered by the path. *4 points*

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