M54-FE-002

Mathematics 54 — Elementary Analysis II

Final Examination — Second Semester, AY 2013 -2014

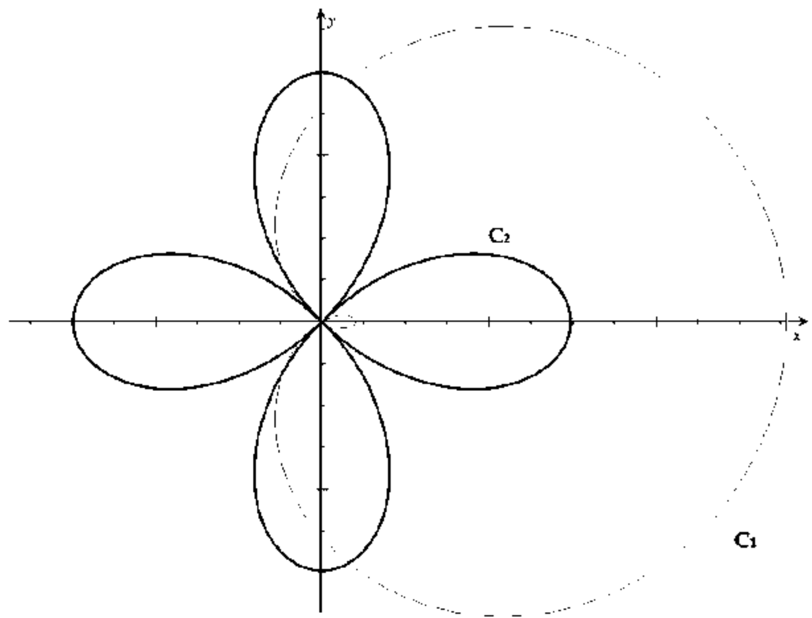
I. TRUE OR FALSE. Write TRUE if the statement is always true, otherwise write FALSE. *(1 pt. each)*

1. If $\vec{u}$ and $\vec{v}$ are unit vectors then $\vec{u} \times \vec{v}$ is also a unit vector.
2. A circle with radius a has a curvature of $\dfrac{1}{a}$.
3. The polar curve $r = \dfrac{6}{2 - \sin\theta}$ is an ellipse.
4. A hyperbola that is rotated about its conjugate axis will form a hyperboloid of 2 sheets.
5. $z = x^3 \sin(2y) + 4y^3$ is differentiable everywhere.

II. PROBLEM SOLVING. Solve the following problems carefully and intelligently. Show all necessary solutions to merit full points.

1. Evaluate the following. *(5 pts. each)*
   (a) $\displaystyle\int 2^x \sin 2x \, dx$
   (b) $\displaystyle\int \frac{2x^2 - x + 8}{x(x^2+4)} \, dx$
2. Given the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{3} = 1$.
   (a) Sketch the graph of the conic section and label all important points. *(3 pts.)*
   (b) Find the equation of the ellipse whose foci and vertices are the vertices and foci, respectively of the given hyperbola. *(2 pts.)*
3. A particle is moving in space with velocity given by $\vec{V(t)} = \langle 4\sin 2t, -6, -4\cos 2t \rangle$, and $\vec{R(0)} = \langle 2, 0, -1 \rangle$. Find the following. *(3 pts. each)*
   (a) The position vector $\vec{R}(t)$ and the acceleration vector $\vec{A}(t)$ of the particle.
   (b) The total distance travelled by the particle from $t = 1$ up to $t = 6$.
   (c) The curvature of the particle's trajectory at any time t.
4. Given the points $A(7, 9, -2)$ and $B(-3, 5, 0)$. Find the following: *(3 pts. each)*
   (a) The equation of the sphere with a diameter whose endpoints are A and B.
   (b) The parametric equations of the line $\overline{AB}$.
5. Find the equation of the tangent plane to the surface $3z^2 = e^{x^2 - y^2} + 2xyz$ at the point $(1, -1, -1)$. *(5 pts.)*
6. Find the distance between the point $(3, 0, 1)$ and the plane $2x - y - 2z = 13$. *(3 pts.)*
7. Let $f(x, y) = \tan(2x - \ln y)$ with $x = \dfrac{2s}{t}$ and $y = 2st + t^2$. Find $\dfrac{\partial f}{\partial s}$ and $\dfrac{\partial f}{\partial t}$. *(6 pts.)*

8. Given the polar curves, $C_1 : r = 2\cos 2\theta$, with $\frac{-\pi}{4} \leq \theta \leq \frac{\pi}{4}$ and $C_2 : r = \sqrt{3} + 2\cos\theta$.



(a) Set-up the definite integral needed to find the area inside C_1 but outside the inner loop of C_2. *(3 pts.)*

(b) Set-up the definite integral needed to find the perimeter of C_2. *(3 pts.)*

9. Find the moving trihedral to the vector-valued function $\vec{R}(t) = \cos^3 t\hat{\imath} + \sin^3 t\hat{\jmath}$. *(6 pts.)*

10. Show that the limit does not exist. *(4 pts.)*

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - 2y}{xy - y + x^2}$$

-END OF EXAM-