M54-LE3-003
Mathematics 54 Elementary Analysis II
Third Long Exam First Semester, AY 2014 -2015

I. TRUE OR FALSE. Write the TRUE if the statement is always true, otherwise write FALSE. *(1 pt. each)*

1. The graph of the surface $\frac{x^2}{25} + \frac{y^2}{4} = z^2 - 1$ is a hyperbolic paraboloid.
2. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ then $\vec{A} = \vec{0}$ or $\vec{B} = \vec{0}$.
3. The surface of revolution formed by revolving the curve $x + z = 2$ about the x-axis has equation $y^2 + z^2 = x^2 - 4x + 4$.
4. The graph of the surface $e^x = \sin(y - 1)$ in $\mathbb{R}^3$ is a cylinder.

II. PROBLEM SOLVING. Do as indicated. Show complete and clear solutions to get full points. Box your final answers.

1. Given the equation
$$x^2 + y^2 = 25z + 50 \ .$$
Identify the type of quadric and find its traces along the coordinate planes. Sketch the graph. *(5 pts.)*

2. Find the equation of the sphere whose endpoints of a diameter are $(1, -5, 4)$ and $(3, 1, 0)$. *(3 pts.)*

3. Find all values of n such that the two vectors $3n\hat{i} + 2\hat{j} - 5\hat{k}$ and $4\hat{i} - n\hat{j} + 3\hat{k}$ are orthogonal. *(2 pts.)*

4. Consider the plane $\pi : 2x - 2y - 3z = 7$ and the line $L : \frac{x-1}{2} = 3 - y = \frac{z-2}{2}$.
   (a) Show that L is parallel to π. *(2 pts.)*
   (b) Find the equation of the plane parallel to π that contains L. *(2 pts.)*

5. Find the volume of the parallelepiped with adjacent sides defined by the vectors $\langle -2, 3, 1\rangle$, $\langle -1, 3, 3\rangle$ and $\langle 0, 4, -2\rangle$. *(5 pts.)*

6. Find the parametric equations of the line L that passes through $(5, 0, -1)$ and is parallel to both the planes $\pi_1 : x + 2y - z = 4$ and $\pi_2 : 3x - y - 2z = 0$. *(5 pts.)*

7. Find the distance of the point $P(1, 3, 1)$ to the line L given by the symmetric equations $x + 1 = 2y = -z - 1$ *(5 pts.)*

8. Find the intersection of the line L having the parametric equations
$$\begin{cases} x &= 3t \\ y &= 2t - 1 \\ z &= 5 - 5t \end{cases}$$
and the plane $\pi : 3y + z = 4x + 2$. *(2 pts.)*

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