M54-LE3-004

Mathematics 54 Elementary Analysis II
Third Long Exam First Semester, AY 2014 -2015

I. True or False. Write TRUE if the statement is always true, otherwise, write FALSE. *1 point each*

1. The graph of the equation $3x^2 - 4y^2 + z^2 + 4 = 0$ is a hyperboloid of two sheets.
2. The graph of the surface $x^2 - z = y^2$ is an elliptic paraboloid.
3. The vector $5\vec{A}$ has magnitude 5 and the same direction as $\vec{A}$.
4. If $\vec{v} \times \vec{u} = \vec{0}$, then $\vec{v} = \vec{0}$ or $\vec{u} = \vec{0}$.

II. Problem Solving. Do as indicated. Show complete and clear solutions to get full points.*9 points each*

1. Let $A(1,-1,0), B(2,0,1), C(5,-1,2), D(-1,0,0)$.
   (a) Find the equation of the sphere whose endpoints of a diameter are A and C.
   (b) Find the symmetric equations of the line containing C and D.
   (c) Find the volume of the parallelepiped having the edges DA, DB and DC.

2. Given the surface: $4 - z = 4x^2 + 4y^2$.
   (a) Identify its traces at the coordinate planes.
   (b) Identify its type as a quadric surface and sketch its graph.
   (c) Find an equation of the curve in the *xz*-plane that if revolved about the *x*-axis will form to the given surface.

3. Given the lines: $\ell_1 : \begin{cases} x = 2t - 1 \\ y = t + 2 \\ z = 3 - t \end{cases}$ and $\ell_2 : \frac{1-x}{3} = y - 3 = \frac{z-2}{2}$.
   (a) Find the intersection of $\ell_1 and \ell_2$.
   (b) Find the equation of the plane π containing both lines.
   (c) Find the distance of ℓ_1 to the point (0,1,2).

4. Given the planes: $\pi_1 : 3x - y + z = 1$ and $\pi_2 : x + 4y - 2z = 0$.
   (a) Find the line of intersection of $\pi_1 and \pi_2$.
   (b) Find the angle between the two given planes.
   Find the distance of π_1 to the point (-1,-2,3).

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