M54-LE4-003

Mathematics 54 Elementary Analysis II
Fourth Long Exam First Semester, AY 2014 -2015

1. Let $\vec{R}(t) = \langle e^{2t}, 2-3t^2, t\ln t \rangle$. Evaluate the following. *4 points each*
   (a) $\lim_{t\to 0^+} \vec{R}(t)$
   (b) $\int_1^2 \vec{R}(t)dt$

2. Let $\vec{R}(t) = \langle 1-t, 1+t^2, t-t^2 \rangle$.
   (a) Find the moving trihedral of $\vec{R}$ at $t = 1$. *8 points*
   (b) Find the equation of the normal plane at $t = 0$. *4 points*

3. Let $\vec{R}(t) = \sqrt{2}t\hat{i} + e^t\hat{j} - e^{-t}\hat{k}$.
   *(a)* Reparametrize $\vec{R}(t)$ with respect to the arc length measured from $t = 0$ in the direction of increasing $t$. (Hint: 2 cosh = $e^t + e^{-t}$) *4 points*
   (b) Find the curvature of $\vec{R}(t)$ at $t = 0$. *3 points*

4. A bug moving in space has a velocity given by $\vec{v}(t) = \langle -e^{2-t}, 2t+1, 1 \rangle$. At $t = 2$, it was found to be at (1,7,1).
   (a) Find the position function of the bug at any time $t$. *3 points*
   (b) Find the normal component of acceleration of the bug at $t = 2$. *4 points*

5. A cannon 20 meters above the ground with its barrel at angle $30^o$ with the horizontal fires a ball with a muzzle speed of 30 meters per second. If acceleration due to gravity is rounded off to 10 $m/s^2$, find:
   (a) the position function at any time $t$ *1 point*
   (b) the maximum height of the cannon ball *2 points*
   (c) the speed at the instant the cannon ball hits the ground *2 points*
   (d) the farthest distance the cannon ball can reach *1 point*

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