M54-LE5-001

Mathematics 54 | Elementary Analysis II

Fifth Long Exam | First Semester, AY 2014 -2015

I. 1. Find and sketch the domain of $f(x,y) = \dfrac{\sqrt{1-xy}}{x+2y}$. *(4 points)*

2. Sketch the contour plot of $g(x,y) = y + (x-1)^2$ using levels $k = -2, 0, 2$. *(3 points)*

3. Find the range of $f(x,y) = 2 + \sqrt{36 - x^2 - y^2}$ *(2 points)*

II. 1. Evaluate $\displaystyle\lim_{(x,y)\to(0,2)} \frac{x^4 - (y-2)^4}{x^2 + (y-2)^2}$. *(2 points)*

2. Consider $F(x,y,z) = \dfrac{6xy^2z}{3x^6 + y^6 + 2z^2}$.

(a) Evaluate $\displaystyle\lim_{(x,y,z)\to(0,0,0)} F(x,y,z)$ using the path defined by $x = t,\ y = t,\ z = t^3$. *(2 points)*

(b) Use another path to show that $\displaystyle\lim_{(x,y,z)\to(0,0,0)} F(x,y,z)$ does not exist. *(2 points)*

III. 1. Given the function

$$f(x,y) = \begin{cases} \dfrac{x^2 \sin(2x) + y^3}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases},$$

show that $f_x(0,0)$ and $f_y(0,0)$ both exist, and determine their values. *(4 points)*

2. By definition, show that the function $g(x,y) = 3x - 5y + 1$ is differentiable everywhere in $\mathbb{R}^2$. *(3 points)*

IV. 1. Show that the function $f(x,y) = e^{2x}\cos 3y$ satisfies the partial differential equation:
$4f + f_{yx} = f_{xx} + 2f_y$ *(4 points)*

2. Let $z = \tan(xy) + x^3$, where $x = 1 - \ln t$ and $y = t^2 - 1$. Use multivariate chain rule to find $\dfrac{dz}{dt}$ at $t = 1$. *(4 points)*

3. Find $\dfrac{\partial z}{\partial x}$ if z is a differentiable function of x and y related by the equation

$$z^x = 4xy^3 - z$$

(3 points)

V. Approximate $e^{0.1}(1.95)^3$ using local linear approximation. *(4 points)*

VI. Write TRUE if the statement is always true. Otherwise, write FALSE. *(1 point each)*

1. If $F(x,y,z)$ is differentiable at (a,b,c), then it is continuous at (a,b,c).
2. For all functions $f(x,y)$, $f_{xy} = f_{yx}$ if f is continuous on its domain.
3. $f(x,y) = \ln(x^2 + y^2)$ is differentiable everywhere on its domain.

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