



Mathematics 54
Fifth Long Exam

M54-LE5-002
Elementary Analysis II
First Semester, AY 2014 -2015

I. Identify and sketch the graph of the domain of f(x, y) = (sqrt(25 - x^2 - y^2)) / (3 - y) on the Cartesian plane. [3 pts]

II. Sketch the level curves of g(x, y) = y/x at k = -2, 0 and 2 on one Cartesian plane. Label each curve according to the corresponding value of k. [3 pts]

III. Evaluate lim (x,y) -> (0,0) (x^4 + x^2y^2 - 2y^4) / (x^2 + 2y^2). [2 pts]

IV. Given that f(x, y) = { 3x sin(y^2) / (x^2 + y^4) if (x, y) != (0, 0); 0 if (x, y) = (0, 0) }

1. Show that lim (x,y) -> (0,0) f(x, y) does not exist. [4 pts]

2. Is f differentiable at (0, 0)? Justify. [1 pt]

V. Let

f(x, y) = { 4xy^2 / (x^2 + y^2) if (x, y) != (1, 1); 2 if (x, y) = (1, 1) }

Using the limit definition of the partial derivative, find f_x(1, 1). [3 pts]

VI. Let f(x, y) = x^y, where x > 0. Find the value of x_0 for which f_xy(x_0, 1) = 2. [4 pts]

VII. Given that dz/dx = sqrt(5xy^2 + 3) ln(x + y + e) and dz/dy | (x,y)=(1,-1) = -4.

1. Use differentials to estimate Delta z as (x, y) changes from (1, -1) to (1.01, -0.99). [3 pts]

2. If x = sinh(2st) + t/2 and y = -te^s/2, evaluate dz/ds when s = 0 and t = 2. [5 pts]

VIII. Suppose z = f(x, y) is a differentiable function defined implicitly by the equation

G(x, y, z) = 2x^2yz^2 - 1

where G is differentiable such that G_x(1, 2, -1) = 2, G_y(1, 2, -1) = -1 and G_z(1, 2, -1) = 1.

Show that dz/dx | (x,y)=(1,2) = 2/3. [3 pts]

IX. Write TRUE if the statement is always true. Otherwise, write FALSE. [1 pt each]

1. If a function z = f(x, y) is differentiable everywhere, then its first partial derivatives exist at all (x, y).

2. There exists a function h such that h_y(x, y, z) = 2xyz and h_z(x, y, z) = xz^2.

3. The range of f(x, y) = x^2 + cos(y) is [-1, +infinity).

4. If f_y(a, b) exists, then lim (x,y) -> (a,b) f(x, y) exists along x=a.