UP SCHOOL OF STATISTICS STUDENT COUNCIL

Education and Research

M54-LE5-003

Mathematics 54 — Elementary Analysis II

Fifth Long Exam — First Semester, AY 2014 -2015

1. Given

$$f(x,y) = \begin{cases} \dfrac{x \sin^{-1}(y+1)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   a. Identify and sketch the domain of f. *(3 pts)*

   b. Show that f has an essential discontinuity at $(0,0)$. *(4 pts)*

2. Given

$$g(x,y) = \begin{cases} \dfrac{x^2-y^2}{x-y} & \text{if } x \neq y; \\ 2x & \text{if } x = y. \end{cases}$$

   Using the <u>limit definition</u> of $g_x(x,y)$, find $g_x(3,3)$. *(3 pts)*

3. Let ρ be a continuous function of a single variable and h a differentiable function of x and y such that

$$h_x(x,y) = \frac{1}{y}\rho\left(\frac{x}{y}\right) \quad \text{and} \quad \frac{-x}{y^2}\rho\left(\frac{x}{y}\right) = h_y(x,y)$$

   with $\rho(2) = 4$, $\rho'(2) = 2$, and $h(2,1) = -5$.

   a. Find $h_{xy}(2,1)$. *(3 pts)*

   b. Use the local linear approximation of h at $(2,1)$ to estimate the value of $h(2.06, 0.97)$. *(3 pts)*

   c. If $x = 6v - u^2$ and $y = \sqrt[3]{u-v}$, find $\dfrac{\partial h}{\partial u}$ when $u = 4$ and $v = 3$. *(4 pts)*

4. The equation

$$xyz = F(x,y,z)$$

   defines z implicitly as a differentiable function of the independent variables x and y. If $F_x(x,y,z) = 0$ and $F_y(x,y,z) = 0$ for all (x,y,z), show that $x\dfrac{\partial z}{\partial x} = y\dfrac{\partial z}{\partial y}$. *(4 pts)*

5. Write TRUE if the statement is true, FALSE if it is false, and **justify your answer** in no more than two sentences. *(2 pts each)*

   a. There is a function f of x and y whose partial derivatives are $f_x(x,y) = x - 2y$ and $f_y(x,y) = 2x + 2y$.

   b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+xy^2}{x^2+y^2} = \lim_{x\to 0} x = 0$

   c. The curve of intersection of $z = x\cos(y)$ and $y = \pi$ has a horizontal tangent line (i.e., parallel to the xy-plane) at the point $(1, \pi, -1)$.

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