UP SCHOOL OF STATISTICS STUDENT COUNCIL
Education and Research
erho.weebly.com erhomyhero@gmail.com /erhoismyhero @erhomyhero
EDUCATION AND RESEARCH

M54-LE5-004

Mathematics 54 | Elementary Analysis II
Fifth Long Exam | Second Semester, AY 2013 -2014

I. Identify and sketch the domain of $f(x,y) = \dfrac{\ln(y^2-x^2)}{\sqrt{1-|y|}}$. *(4 points)*

II. Given: $f(x,y) = \begin{cases} \dfrac{2x^2\sin(x)+y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

1. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist. *(4 points)*
2. Using the definition of partial derivatives, determine $f_x(0,0)$. *(4 points)*
3. Is f differentiable at $(0,0)$? Justify. *(1 point)*

III. Let z be a differentiable function of x and y such that $\dfrac{\partial z}{\partial x} = \sqrt[3]{2-xy}\,e^{2x+3y}$ and $\left.\dfrac{\partial z}{\partial y}\right|_{\substack{x=3\\y=-2}} = -6$.

1. Find $\dfrac{\partial^2 z}{\partial y \partial x}$. *(3 points)*
2. Use differentials to approximate the change in z, i.e. Δz, as (x,y) changes from $(3,-2)$ to $(2.99,-1.95)$. *(3 points)*
3. If $x = 3u + 4\ln(2v-7)$ and $y = 2u\sinh(4u-v) - \dfrac{v}{2}$, find $\dfrac{\partial z}{\partial v}$ at $u=1$ and $v=4$. *(6 points)*

IV. Suppose F is a differentiable function of x, y, and z, with $F_x(1,1,2)=4$, $F_y(1,1,2)=-8$, and $F_z(1,1,2)=4$. In addition, suppose z is a differentiable function of x and y implicitly defined in the equation $F(x,y,z)=2$.

1. Show that $\dfrac{\partial z}{\partial x} = -1$ and $\dfrac{\partial z}{\partial y} = 2$ at $(1,1)$. *(2 points)*
2. Use (a) to find the equation of the tangent plane to the graph of $z=f(x,y)$ at the point $(1,1,2)$ on the graph. *(2 points)*

V. Find/Evaluate the following.

1. the level curves of $f(x,y) = 3 - x^2 + y$ at $k=4$ (Give the equation and identify the type of curve.) *(1 point)*
2. the range of $g(x,y) = x^2+y^2+3$ *(1 point)*
3. $\displaystyle\lim_{(x,y)\to(5,0)} \frac{2x[(x-5)^2+y^2]-7(x-5)^2-7y^2}{3(x-5)^2+3y^2}$ *(2 points)*
4. the constant a such that f is a polynomial function of x and y with $f_x(x,y) = y^2+6y+x$ and $f_y(x,y) = ax+2xy$ *(2 points)*

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